The Quadratic Formula: answers

Worked solutions and mark scheme · GCSE Maths · Higher tier

M = method mark, A = accuracy mark (needs the method), B = independent mark, ft = follow through from an earlier wrong answer.

Level 1 · Warm-up

13 marks

Substituting into the formula, with equations already in the form $ax^2 + bx + c = 0$.

1 Write down the values of a, b and c you would use in the quadratic formula to solve $3x^2 - 5x + 1 = 0$.

Answer: $a = 3,\ b = -5,\ c = 1$

$3x^2 - 5x + 1 = 0$	*Compare with $ax^2 + bx + c = 0$*
$a = 3, \quad b = -5, \quad c = 1$	*Keep the sign with each number*

Marks B1 All three correct, including $b = -5$

2 Solve $x^2 + 6x + 4 = 0$. Give your answers in surd form.

Answer: $x = -3 \pm \sqrt{5}$

$a = 1, \quad b = 6, \quad c = 4$	
$x = \dfrac{-6 \pm \sqrt{6^2 - 4 \times 1 \times 4}}{2 \times 1}$	*Substitute into the formula*
$= \dfrac{-6 \pm \sqrt{20}}{2}$	$36 - 16 = 20$
$= \dfrac{-6 \pm 2\sqrt{5}}{2} = -3 \pm \sqrt{5}$	$\sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$

Marks M1 Correct substitution: $\dfrac{-6 \pm \sqrt{6^2 - 4 \times 1 \times 4}}{2 \times 1}$ M1 $\dfrac{-6 \pm \sqrt{20}}{2}$ A1 $-3 \pm \sqrt{5}$ — allow $\dfrac{-6 \pm \sqrt{20}}{2}$ for M1 M1 only

3 Solve $x^2 - 4x - 3 = 0$. Give your answers correct to 2 decimal places.

Answer: $x = 4.65$ or $x = -0.65$

$a = 1, \quad b = -4, \quad c = -3$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times (-3)}}{2 \times 1}$$

Use brackets around negative numbers

$$= \frac{4 \pm \sqrt{28}}{2}$$

$16 + 12 = 28$*: minus a negative is a plus*

$x = 4.645\ldots$ or $x = -0.645\ldots$

$x = 4.65$ or $x = -0.65$

Round at the end

Marks M1 Correct substitution: $\frac{4 \pm \sqrt{(-4)^2 - 4 \times 1 \times (-3)}}{2}$ M1 $\frac{4 \pm \sqrt{28}}{2}$ A1 4.65 and -0.65

4 Solve $2x^2 + 3x - 1 = 0$. Give your answers correct to 2 decimal places.

Answer: $x = 0.28$ or $x = -1.78$

$a = 2, \quad b = 3, \quad c = -1$

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-1)}}{2 \times 2}$$

$$= \frac{-3 \pm \sqrt{17}}{4}$$

$9 + 8 = 17$*, and the bottom is* $2a = 4$

$x = 0.280\ldots$ or $x = -1.780\ldots$

$x = 0.28$ or $x = -1.78$

Marks M1 Correct substitution: $\frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-1)}}{2 \times 2}$ M1 $\frac{-3 \pm \sqrt{17}}{4}$ A1 0.28 and -1.78

5 Solve $x^2 + 2x - 5 = 0$. Give your answers correct to 2 decimal places.

Answer: $x = 1.45$ or $x = -3.45$

$a = 1, \quad b = 2, \quad c = -5$

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times (-5)}}{2 \times 1}$$

$$= \frac{-2 \pm \sqrt{24}}{2}$$

$4 + 20 = 24$

$x = 1.449\ldots$ or $x = -3.449\ldots$

$x = 1.45$ or $x = -3.45$

Marks M1 Correct substitution into the formula M1 $\frac{-2 \pm \sqrt{24}}{2}$ A1 1.45 and -3.45

Level 2 · Build 15 marks

Rearranging first, negative coefficients and exact surd answers.

6 Solve $3x^2 = 7x - 1$. Give your answers correct to 2 decimal places.

Answer: $x = 2.18$ or $x = 0.15$

$3x^2 - 7x + 1 = 0$ *Rearrange so one side is 0 first*

$a = 3, \quad b = -7, \quad c = 1$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4 \times 3 \times 1}}{2 \times 3} = \frac{7 \pm \sqrt{37}}{6}$$

$x = 2.180\ldots$ or $x = 0.152\ldots$

$x = 2.18$ or $x = 0.15$

Marks M1 $3x^2 - 7x + 1 = 0$ M1 Correct substitution: $\frac{7 \pm \sqrt{49 - 12}}{6}$ A1 2.18 and 0.15

7 Solve $x^2 - 6x + 2 = 0$. Give your answers in the form $a \pm \sqrt{b}$.

Answer: $x = 3 \pm \sqrt{7}$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4 \times 1 \times 2}}{2} \qquad a = 1, b = -6, c = 2$$

$$= \frac{6 \pm \sqrt{28}}{2} \qquad 36 - 8 = 28$$

$$= \frac{6 \pm 2\sqrt{7}}{2} \qquad \sqrt{28} = \sqrt{4 \times 7} = 2\sqrt{7}$$

$$= 3 \pm \sqrt{7} \qquad \textit{Divide both terms on top by } 2$$

Marks M1 Correct substitution into the formula M1 $\frac{6 \pm \sqrt{28}}{2}$ A1 $3 \pm \sqrt{7}$

8 Solve $5x^2 + 2x - 4 = 0$. Give your answers correct to 2 decimal places.

Answer: $x = 0.72$ or $x = -1.12$

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 5 \times (-4)}}{2 \times 5}$$

$$= \frac{-2 \pm \sqrt{84}}{10} \qquad 4 + 80 = 84$$

$$x = 0.716\ldots \quad \text{or} \quad x = -1.116\ldots$$

$$x = 0.72 \quad \text{or} \quad x = -1.12$$

Marks M1 Correct substitution into the formula M1 $\frac{-2 \pm \sqrt{84}}{10}$ A1 0.72 and -1.12

9 Solve $x(x + 4) = 7$. Give your answers correct to 2 decimal places.

Answer: $x = 1.32$ or $x = -5.32$

$$x^2 + 4x - 7 = 0 \qquad \textit{Expand and make one side } 0$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times (-7)}}{2} = \frac{-4 \pm \sqrt{44}}{2}$$

$$x = 1.316\ldots \quad \text{or} \quad x = -5.316\ldots$$

$$x = 1.32 \quad \text{or} \quad x = -5.32$$

Marks M1 $x^2 + 4x - 7 = 0$ M1 Correct substitution: $\frac{-4 \pm \sqrt{16 + 28}}{2}$ A1 1.32 and -5.32

10 Solve $2x^2-4x-3=0$. Give your answers in the form $\frac{a \pm \sqrt{b}}{c}$, where a, b and c are whole numbers.

Answer: $x = \frac{2 \pm \sqrt{10}}{2}$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4 \times 2 \times (-3)}}{2 \times 2}$$

$$= \frac{4 \pm \sqrt{40}}{4} \qquad 16 + 24 = 40$$

$$= \frac{4 \pm 2\sqrt{10}}{4} \qquad \sqrt{40} = \sqrt{4 \times 10} = 2\sqrt{10}$$

$$= \frac{2 \pm \sqrt{10}}{2} \qquad \textit{Divide every term by } 2$$

Marks M1 Correct substitution: $\frac{4 \pm \sqrt{16+24}}{4}$ M1 $\frac{4 \pm \sqrt{40}}{4}$ A1 $\frac{2 \pm \sqrt{10}}{2}$

Level 3 · Exam-style — 20 marks

Show that, contexts and spotting mistakes, in real GCSE style.

11 (a) A rectangle has length $(x + 3)$ cm and width $(x - 1)$ cm. Its area is 20 cm^2. Show that $x^2 + 2x - 23 = 0$.

Answer: $(x + 3)(x - 1) = 20$, so $x^2 + 2x - 3 = 20$, so $x^2 + 2x - 23 = 0$.

$$(x + 3)(x - 1) = 20 \qquad \textit{Area} = \textit{length} \times \textit{width}$$

$$x^2 + 2x - 3 = 20$$

$$x^2 + 2x - 23 = 0$$

Marks M1 $(x + 3)(x - 1) = 20$ A1 Correct expansion and rearrangement to $x^2 + 2x - 23 = 0$

(b) Solve $x^2 + 2x - 23 = 0$ to find the value of x. Give your answer correct to 2 decimal places.

Answer: $x = 3.90$

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times (-23)}}{2} = \frac{-2 \pm \sqrt{96}}{2}$$

$$x = 3.898\ldots \quad \text{or} \quad x = -5.898\ldots$$

$$x = 3.90 \qquad \textit{A negative } x \textit{ would make the width negative, so reject it}$$

Marks M1 Correct substitution: $\frac{-2 \pm \sqrt{4+92}}{2}$ A1 3.90 B1 Rejects the negative solution, because $x-1$ would be negative

12 Hari uses the quadratic formula to solve $x^2 - 3x - 5 = 0$. He writes $x = \dfrac{-3 \pm \sqrt{9 - 20}}{2}$. Explain the two mistakes Hari has made, and solve the equation correctly, giving your answers to 2 decimal places.

Answer: $-b$ should be $+3$, and $-4ac = -4 \times 1 \times (-5) = +20$, so the square root is $\sqrt{29}$. $x = 4.19$ or $x = -1.19$.

$a = 1, \quad b = -3, \quad c = -5$

$-b = -(-3) = 3$ — *Mistake 1: Hari used* -3

$b^2 - 4ac = 9 - 4 \times 1 \times (-5) = 29$ — *Mistake 2: minus a negative is a plus, so* $+20$

$x = \dfrac{3 \pm \sqrt{29}}{2} = 4.192\ldots$ or $-1.192\ldots$

$x = 4.19$ or $x = -1.19$

Marks B1 Explains that $-b = +3$, because $b = -3$ B1 Explains that $b^2 - 4ac = 9 + 20 = 29$, because c is negative B1 4.19 and -1.19

13 Solve $4x^2 - 12x + 7 = 0$. Give your answers in their simplest exact form.

Answer: $x = \dfrac{3 \pm \sqrt{2}}{2}$

$x = \dfrac{12 \pm \sqrt{(-12)^2 - 4 \times 4 \times 7}}{2 \times 4}$

$= \dfrac{12 \pm \sqrt{32}}{8}$ — $144 - 112 = 32$

$= \dfrac{12 \pm 4\sqrt{2}}{8}$ — $\sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$

$= \dfrac{3 \pm \sqrt{2}}{2}$ — *Divide every term by* 4

Marks M1 Correct substitution: $\dfrac{12 \pm \sqrt{144 - 112}}{8}$ M1 $\dfrac{12 \pm \sqrt{32}}{8}$ or $\dfrac{12 \pm 4\sqrt{2}}{8}$ A1 $\dfrac{3 \pm \sqrt{2}}{2}$

14 (a) Show that the equation $\frac{2}{x}+\frac{1}{x+1}=1$ can be written as $x^2-2x-2=0$.

Answer: $2(x+1)+x=x(x+1)$, so $3x+2=x^2+x$, so $x^2-2x-2=0$.

$2(x+1)+x=x(x+1)$ *Multiply every term by $x(x+1)$*

$3x+2=x^2+x$

$0=x^2-2x-2$

Marks M1 Multiplies by $x(x+1)$: $2(x+1)+x=x(x+1)$ M1 $3x+2=x^2+x$ A1 Correct rearrangement to $x^2-2x-2=0$

(b) Hence solve $\frac{2}{x}+\frac{1}{x+1}=1$. Give your answers in the form $a\pm\sqrt{b}$.

Answer: $x=1\pm\sqrt{3}$

$x=\frac{2\pm\sqrt{(-2)^2-4\times1\times(-2)}}{2}=\frac{2\pm\sqrt{12}}{2}$

$=\frac{2\pm2\sqrt{3}}{2}=1\pm\sqrt{3}$ *$\sqrt{12}=2\sqrt{3}$*

Marks M1 $\frac{2\pm\sqrt{4+8}}{2}$ A1 $1\pm\sqrt{3}$

15 The height, h metres, of a ball t seconds after it is thrown is $h=2+20t-5t^2$. Find the two times when the ball is 10 m high. Give your answers correct to 2 decimal places.

Answer: $t=0.45$ s and $t=3.55$ s

$2+20t-5t^2=10$ *Set the height equal to 10*

$5t^2-20t+8=0$

$t=\frac{20\pm\sqrt{(-20)^2-4\times5\times8}}{2\times5}=\frac{20\pm\sqrt{240}}{10}$

$t=0.450\ldots$ or $t=3.549\ldots$

$t=0.45$ s and $t=3.55$ s *Once on the way up, once on the way down*

Marks M1 $5t^2-20t+8=0$ oe M1 Correct substitution into the formula A1 0.45 A1 3.55

Level 4 · Challenge 20 marks
Grade 8–9: fractions, Pythagoras, unknown coefficients and exact answers.

16 Show that the solutions of $x^2 + 8x + 11 = 0$ can be written as $x = -4 \pm \sqrt{5}$.

Answer: $x = \dfrac{-8 \pm \sqrt{20}}{2} = \dfrac{-8 \pm 2\sqrt{5}}{2} = -4 \pm \sqrt{5}$

$$x = \frac{-8 \pm \sqrt{8^2 - 4 \times 1 \times 11}}{2}$$

$$= \frac{-8 \pm \sqrt{20}}{2} \qquad 64 - 44 = 20$$

$$= \frac{-8 \pm 2\sqrt{5}}{2} \qquad \textit{"Show that": every step must be written down}$$

$$= -4 \pm \sqrt{5}$$

Marks M1 Correct substitution: $\dfrac{-8 \pm \sqrt{64 - 44}}{2}$ M1 $\sqrt{20} = 2\sqrt{5}$ seen A1 Fully correct working to $-4 \pm \sqrt{5}$

17 (a) A right-angled triangle has shorter sides x cm and $(x + 2)$ cm, and hypotenuse $(2x - 1)$ cm. Show that $2x^2 - 8x - 3 = 0$.

Answer: $x^2 + (x + 2)^2 = (2x - 1)^2$, so $2x^2 + 4x + 4 = 4x^2 - 4x + 1$, so $2x^2 - 8x - 3 = 0$.

$$x^2 + (x + 2)^2 = (2x - 1)^2 \qquad \textit{Pythagoras: } a^2 + b^2 = c^2$$

$$x^2 + x^2 + 4x + 4 = 4x^2 - 4x + 1$$

$$2x^2 + 4x + 4 = 4x^2 - 4x + 1$$

$$0 = 2x^2 - 8x - 3 \qquad \textit{Move everything to the right-hand side}$$

Marks M1 $x^2 + (x + 2)^2 = (2x - 1)^2$ M1 Both sides expanded correctly: $2x^2 + 4x + 4 = 4x^2 - 4x + 1$ A1 Correct rearrangement to $2x^2 - 8x - 3 = 0$

(b) Work out the length of the hypotenuse. Give your answer correct to 2 decimal places.

Answer: 7.69 cm

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4 \times 2 \times (-3)}}{2 \times 2} = \frac{8 \pm \sqrt{88}}{4}$$

$x = 4.345\ldots$ (or $-0.345\ldots$) *A length can't be negative*

$2x - 1 = 2 \times 4.345\ldots - 1 = 7.690\ldots$ *Use the unrounded value of x*

hypotenuse $= 7.69$ cm

Marks M1 Correct substitution: $\dfrac{8 \pm \sqrt{64 + 24}}{4}$ A1 $x = 4.345\ldots$, rejecting the negative solution A1 7.69 cm

18 The equation $x^2 + px + 7 = 0$ has solutions $x = -3 \pm \sqrt{2}$. Find the value of p.

Answer: $p = 6$

$$x = \frac{-p \pm \sqrt{p^2 - 28}}{2}$$ *The formula with $a = 1, b = p, c = 7$*

$$-\frac{p}{2} = -3$$ *Match the part outside the square root with -3*

$$p = 6$$

$$\frac{\sqrt{36 - 28}}{2} = \frac{\sqrt{8}}{2}$$ *Check: the surd part should match $\sqrt{2}$ too*

$$= \frac{2\sqrt{2}}{2} = \sqrt{2}$$

Marks M1 $x = \frac{-p \pm \sqrt{p^2 - 28}}{2}$, or $(x - (-3 + \sqrt{2}))(x - (-3 - \sqrt{2}))$ M1 $-\frac{p}{2} = -3$, or expands to $(x + 3)^2 - 2$ A1 $p = 6$

19 Solve $\frac{2x + 1}{x - 2} = x + 3$. Give your answers correct to 2 decimal places.

Answer: $x = 3.19$ or $x = -2.19$

$$2x + 1 = (x + 3)(x - 2)$$ *Multiply both sides by $(x - 2)$*

$$2x + 1 = x^2 + x - 6$$

$$x^2 - x - 7 = 0$$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4 \times 1 \times (-7)}}{2} = \frac{1 \pm \sqrt{29}}{2}$$

$$x = 3.19 \quad \text{or} \quad x = -2.19$$

Marks M1 $2x + 1 = (x + 3)(x - 2)$ M1 $x^2 - x - 7 = 0$ M1 Correct substitution: $\frac{1 \pm \sqrt{1 + 28}}{2}$ — ft their quadratic A1 3.19 and -2.19

20 A rectangle is 3 cm longer than it is wide. Its diagonal is 10 cm long. Work out the width of the rectangle. Give your answer correct to 2 decimal places.

Answer: 5.41 cm

$x^2 + (x+3)^2 = 10^2$ — *Let the width be x. The diagonal makes a right-angled triangle*

$x^2 + x^2 + 6x + 9 = 100$

$2x^2 + 6x - 91 = 0$

$x = \dfrac{-6 \pm \sqrt{6^2 - 4(2)(-91)}}{2(2)}$ — $4(2)(-91)$ *means* $4 \times 2 \times (-91)$

$= \dfrac{-6 \pm \sqrt{764}}{4}$ — $36 + 728 = 764$

$x = 5.410\ldots$ — *Reject the negative solution: a width can't be negative*

width $= 5.41$ cm

Marks M1 $x^2+(x+3)^2 = 10^2$, where x is the width M1 $2x^2+6x-91 = 0$ M1 Correct substitution: $\dfrac{-6 \pm \sqrt{36+728}}{4}$ A1 5.41 cm