Solving Quadratics by Factorising: answers

Worked solutions and mark scheme · GCSE Maths · Higher tier

M = method mark, A = accuracy mark (needs the method), B = independent mark, ft = follow through from an earlier wrong answer.

Level 1 · Warm-up 12 marks

Equations that are already $= 0$ and ready to factorise.

1 Solve $(x - 3)(x + 5) = 0$.

Answer: $x = 3$ or $x = -5$

$x - 3 = 0$ or $x + 5 = 0$

$x = 3$ or $x = -5$

If two brackets multiply to 0, one of them is 0

Marks B1 $x = 3$ B1 $x = -5$

2 Solve $x^2 + 7x + 12 = 0$.

Answer: $x = -3$ or $x = -4$

$(x + 3)(x + 4) = 0$

$x + 3 = 0$ or $x + 4 = 0$

$x = -3$ or $x = -4$

$3 \times 4 = 12$ *and* $3 + 4 = 7$

Marks M1 $(x \pm 3)(x \pm 4)$ A1 $(x + 3)(x + 4) = 0$ A1 $x = -3$ and $x = -4$

3 Solve $x^2 - 6x + 8 = 0$.

Answer: $x = 2$ or $x = 4$

$(x - 2)(x - 4) = 0$

$x = 2$ or $x = 4$

$(-2) \times (-4) = 8$ *and* $(-2) + (-4) = -6$

The signs change: $x - 2 = 0$ *gives* $x = 2$

Marks M1 $(x \pm 2)(x \pm 4)$ A1 $(x - 2)(x - 4) = 0$ A1 $x = 2$ and $x = 4$

4 Solve $x^2 - 5x = 0$.

Answer: $x = 0$ or $x = 5$

$x(x - 5) = 0$

$x = 0$ or $x = 5$

Take out the common factor x. Don't divide by x: you would lose $x = 0$

Marks M1 $x(x - 5) = 0$ A1 $x = 0$ and $x = 5$

5 Solve $x^2 - 36 = 0$.

Answer: $x = 6$ or $x = -6$

$(x + 6)(x - 6) = 0$

$x = -6$ or $x = 6$

Difference of two squares

Both 6^2 and $(-6)^2$ are 36

Marks M1 $(x + 6)(x - 6) = 0$ or $x^2 = 36$ A1 $x = 6$ and $x = -6$

Level 2 · Build

16 marks

Harder signs, repeated roots, and a first look at $ax^2 + bx + c = 0$ and rearranging.

6 Solve $x^2 + 2x - 24 = 0$.

Answer: $x = 4$ or $x = -6$

$(x + 6)(x - 4) = 0$

$x = -6$ or $x = 4$

$6 \times (-4) = -24$ *and* $6 + (-4) = 2$

Marks M1 $(x \pm 6)(x \pm 4)$ A1 $(x + 6)(x - 4) = 0$ A1 $x = 4$ and $x = -6$

7 Solve $x^2 - 10x + 25 = 0$.

Answer: $x = 5$

$(x - 5)(x - 5) = 0$

$x = 5$

$(-5) \times (-5) = 25$ *and* $(-5) + (-5) = -10$

Both brackets give the same answer, so there is only one solution

Marks M1 $(x - 5)(x - 5) = 0$ or $(x - 5)^2 = 0$ A1 $x = 5$ (only)

8 Solve $x^2 + 9x = 0$.

Answer: $x = 0$ or $x = -9$

$x(x + 9) = 0$ *Common factor* x

$x = 0$ or $x = -9$

Marks M1 $x(x + 9) = 0$ A1 $x = 0$ and $x = -9$

9 Solve $x^2 - 3x - 40 = 0$.

Answer: $x = 8$ or $x = -5$

$(x - 8)(x + 5) = 0$ $(-8) \times 5 = -40$ *and* $(-8) + 5 = -3$

$x = 8$ or $x = -5$

Marks M1 $(x \pm 8)(x \pm 5)$ A1 $(x - 8)(x + 5) = 0$ A1 $x = 8$ and $x = -5$

10 Solve $2x^2 + 5x - 3 = 0$.

Answer: $x = \frac{1}{2}$ or $x = -3$

$ac = -6$; $\quad 6 \times (-1) = -6,\ 6 + (-1) = 5$ *Two numbers that multiply to* ac *and add to* b

$2x^2 + 6x - x - 3 = 0$

$2x(x + 3) - 1(x + 3) = 0$

$(2x - 1)(x + 3) = 0$

$x = \frac{1}{2}$ or $x = -3$ $2x - 1 = 0$ *gives* $x = \frac{1}{2}$

Marks M1 $(2x \pm 1)(x \pm 3)$, or a correct split of the middle term A1 $(2x - 1)(x + 3) = 0$ A1 $x = \frac{1}{2}$ and $x = -3$ — allow 0.5

11 Solve $x^2 = 4x + 21$.

Answer: $x = 7$ or $x = -3$

$x^2 - 4x - 21 = 0$ *Make one side* 0 *first*

$(x - 7)(x + 3) = 0$

$x = 7$ or $x = -3$

Marks M1 $x^2 - 4x - 21 = 0$ M1 $(x - 7)(x + 3) = 0$ A1 $x = 7$ and $x = -3$

Level 3 · Exam-style 21 marks
Show that, hence and graph questions in real GCSE style.

12 (a) Factorise $x^2 - 2x - 15$.

Answer: $(x - 5)(x + 3)$

$(-5) \times 3 = -15, \quad (-5) + 3 = -2$

$x^2 - 2x - 15 = (x - 5)(x + 3)$

Marks M1 $(x \pm 5)(x \pm 3)$ A1 $(x - 5)(x + 3)$

(b) Hence solve $x^2 - 2x - 15 = 0$.

Answer: $x = 5$ or $x = -3$

$(x - 5)(x + 3) = 0$ *Use part (a)*

$x = 5$ or $x = -3$

Marks B1 $x = 5$ and $x = -3$ — ft their (a)

13 (a) A rectangle is x cm wide and $(x + 5)$ cm long. Its area is 36 cm^2. Show that $x^2 + 5x - 36 = 0$.

Answer: $x(x + 5) = 36$, so $x^2 + 5x = 36$, so $x^2 + 5x - 36 = 0$.

$x(x + 5) = 36$ *Area = width × length*

$x^2 + 5x = 36$

$x^2 + 5x - 36 = 0$ *Subtract 36 from both sides*

Marks M1 $x(x + 5) = 36$ A1 Expands and rearranges correctly to $x^2 + 5x - 36 = 0$

(b) Solve $x^2 + 5x - 36 = 0$.

Answer: $x = 4$ or $x = -9$

$(x + 9)(x - 4) = 0$ $9 \times (-4) = -36$ *and* $9 + (-4) = 5$

$x = -9$ or $x = 4$

Marks M1 $(x \pm 9)(x \pm 4)$ A1 $(x + 9)(x - 4) = 0$ A1 $x = 4$ and $x = -9$

(c) Write down the width of the rectangle.

Answer: 4 cm

$x = 4$ cm	*A width can't be negative, so reject* $x = -9$

Marks B1 4 cm — ft a positive solution from (b)

14 Ella solves $x^2 - 7x + 10 = 0$. She writes $(x - 2)(x - 5) = 0$, so $x = -2$ or $x = -5$. Explain Ella's mistake, and write down the correct solutions.

Answer: $x = 2$ or $x = 5$. Ella's brackets are right, but $x - 2 = 0$ gives $x = 2$, not -2.

$(x - 2)(x - 5) = 0$	*Ella's factorising is correct*
$x - 2 = 0 \Rightarrow x = 2$	*Add 2 to both sides: the sign changes*
$x - 5 = 0 \Rightarrow x = 5$	

Marks B1 Explains that $x - 2 = 0$ gives $x = 2$ (the signs should change) B1 $x = 2$ and $x = 5$

15 The graph of $y = x^2 - x - 12$ crosses the x-axis at two points. Use algebra to find the coordinates of these points.

Answer: $(4,\ 0)$ and $(-3,\ 0)$

$x^2 - x - 12 = 0$	*On the x-axis, $y = 0$*
$(x - 4)(x + 3) = 0$	
$x = 4$ or $x = -3$	
$(4,\ 0)$ and $(-3,\ 0)$	*Give coordinates, not just the x values*

Marks M1 $x^2 - x - 12 = 0$ M1 $(x - 4)(x + 3) = 0$ A1 $(4,\ 0)$ and $(-3,\ 0)$

16 Solve $3x^2 - 11x - 4 = 0$.

Answer: $x = 4$ or $x = -\frac{1}{3}$

$ac = -12; \quad (-12) \times 1 = -12,\ -12 + 1 = -11$

$3x^2 - 12x + x - 4 = 0$

$3x(x - 4) + 1(x - 4) = 0$

$(3x + 1)(x - 4) = 0$

$x = -\frac{1}{3}$ or $x = 4$

Marks M1 $(3x \pm 1)(x \pm 4)$, or a correct split of the middle term A1 $(3x + 1)(x - 4) = 0$ A1 $x = 4$ and $x = -\frac{1}{3}$

17 Solve $(x + 2)(x - 1) = 4x + 2$.

Answer: $x = 4$ or $x = -1$

$x^2 + x - 2 = 4x + 2$

Expand the brackets first. You can't set each bracket to $4x + 2$

$x^2 - 3x - 4 = 0$

Make one side 0

$(x - 4)(x + 1) = 0$

$x = 4$ or $x = -1$

Marks M1 Expands $(x + 2)(x - 1)$ to $x^2 + x - 2$ M1 $x^2 - 3x - 4 = 0$ M1 $(x - 4)(x + 1) = 0$ A1 $x = 4$ and $x = -1$

Level 4 · Challenge 16 marks

Grade 8–9: fractions, unknown coefficients and contexts with conditions.

18 Solve $\frac{5}{x+2}+\frac{3}{x}=2$.

Answer: $x=3$ or $x=-1$

$5x+3(x+2)=2x(x+2)$	*Multiply every term by* $x(x+2)$
$8x+6=2x^2+4x$	
$2x^2-4x-6=0$	
$x^2-2x-3=0$	*Divide by* 2
$(x-3)(x+1)=0$	
$x=3$ or $x=-1$	

Marks M1 Multiplies by $x(x+2)$: $5x+3(x+2)=2x(x+2)$ M1 $2x^2-4x-6=0$ or $x^2-2x-3=0$ M1 $(x-3)(x+1)=0$ — ft their quadratic A1 $x=3$ and $x=-1$

19 The equation $x^2+bx+c=0$ has solutions $x=4$ and $x=-7$. Find the value of b and the value of c.

Answer: $b=3,\ c=-28$

$(x-4)(x+7)=0$	*Work backwards:* $x=4$ *comes from the bracket* $(x-4)$
$x^2+7x-4x-28=x^2+3x-28$	
$b=3,\quad c=-28$	

Marks M1 $(x-4)(x+7)=0$ M1 Expands to $x^2+3x-28$ A1 $b=3$ and $c=-28$

20 (a) A triangle has base $(x+1)$ cm and perpendicular height $(2x-3)$ cm. Its area is 9 cm^2. Show that $2x^2-x-21=0$.

Answer: $\frac{1}{2}(x+1)(2x-3)=9$, so $2x^2-x-3=18$, so $2x^2-x-21=0$.

$\frac{1}{2}(x+1)(2x-3)=9$	*Area of a triangle* $=\frac{1}{2}\times$ *base* $\times$ *height*
$(x+1)(2x-3)=18$	*Multiply both sides by* 2
$2x^2-3x+2x-3=18$	
$2x^2-x-21=0$	

Marks M1 $\frac{1}{2}(x+1)(2x-3)=9$ M1 $(x+1)(2x-3)=18$, or $2x^2-x-3$ seen A1 Fully correct working to $2x^2-x-21=0$

(b) Work out the length of the base of the triangle.

Answer: 4.5 cm

$ac = -42; \quad (-7) \times 6 = -42,\ -7 + 6 = -1$

$2x^2 - 7x + 6x - 21 = 0$

$x(2x - 7) + 3(2x - 7) = 0$

$(2x - 7)(x + 3) = 0$

$x = 3.5$

$x = -3$ would make the base and height negative

base $= 3.5 + 1 = 4.5$ cm

The question asks for the base, not x

Marks M1 $(2x - 7)(x + 3) = 0$ A1 $x = 3.5$, rejecting $x = -3$ A1 4.5 cm

21 Solve $(x - 3)^2 = 2(x - 3)$.

Answer: $x = 3$ or $x = 5$

$(x - 3)^2 - 2(x - 3) = 0$

Don't divide by $(x - 3)$: it could be 0, and you would lose a solution

$(x - 3)\big[(x - 3) - 2\big] = 0$

Take out the common factor $(x - 3)$

$(x - 3)(x - 5) = 0$

$x = 3 \quad$ or $\quad x = 5$

Marks M1 $(x - 3)^2 - 2(x - 3) = 0$, or $x^2 - 8x + 15 = 0$ M1 $(x - 3)(x - 5) = 0$ A1 $x = 3$ and $x = 5$