# Solving Quadratics by Factorising: answers

Worked solutions and mark scheme · GCSE Maths · Foundation tier

M = method mark, A = accuracy mark (needs the method), B = independent mark, ft = follow through from an earlier wrong answer.

**Level 1 · Warm-up** 12 marks

Equations that are already $= 0$ and ready to factorise.

**1** Solve $(x - 3)(x + 5) = 0$.

**Answer:** $x = 3$ or $x = -5$

$x - 3 = 0$ or $x + 5 = 0$

$x = 3$ or $x = -5$

*If two brackets multiply to $0$, one of them is $0$*

**Marks** B1 $x = 3$ B1 $x = -5$

**2** Solve $x^2 + 7x + 12 = 0$.

**Answer:** $x = -3$ or $x = -4$

$(x + 3)(x + 4) = 0$

$x + 3 = 0$ or $x + 4 = 0$

$x = -3$ or $x = -4$

$3 \times 4 = 12$ *and* $3 + 4 = 7$

**Marks** M1 $(x \pm 3)(x \pm 4)$ A1 $(x + 3)(x + 4) = 0$ A1 $x = -3$ and $x = -4$

**3** Solve $x^2 - 6x + 8 = 0$.

**Answer:** $x = 2$ or $x = 4$

$(x - 2)(x - 4) = 0$

$x = 2$ or $x = 4$

$(-2) \times (-4) = 8$ *and* $(-2) + (-4) = -6$

*The signs change:* $x - 2 = 0$ *gives* $x = 2$

**Marks** M1 $(x \pm 2)(x \pm 4)$ A1 $(x - 2)(x - 4) = 0$ A1 $x = 2$ and $x = 4$

**4** Solve $x^2 - 5x = 0$.

**Answer:** $x = 0$ or $x = 5$

$x(x - 5) = 0$

$x = 0$ or $x = 5$

*Take out the common factor $x$. Don't divide by $x$: you would lose $x = 0$*

**Marks** M1 $x(x - 5) = 0$ A1 $x = 0$ and $x = 5$

**5** Solve $x^2 - 36 = 0$.

**Answer:** $x = 6$ or $x = -6$

$(x + 6)(x - 6) = 0$

$x = -6$ or $x = 6$

*Difference of two squares*

*Both $6^2$ and $(-6)^2$ are $36$*

**Marks** M1 $(x + 6)(x - 6) = 0$ or $x^2 = 36$ A1 $x = 6$ and $x = -6$

## Level 2 · Build

10 marks

Harder signs, repeated roots, and a first look at $ax^2 + bx + c = 0$ and rearranging.

**6** Solve $x^2 + 2x - 24 = 0$.

**Answer:** $x = 4$ or $x = -6$

$(x + 6)(x - 4) = 0$

$x = -6$ or $x = 4$

$6 \times (-4) = -24$ *and* $6 + (-4) = 2$

**Marks** M1 $(x \pm 6)(x \pm 4)$ A1 $(x + 6)(x - 4) = 0$ A1 $x = 4$ and $x = -6$

**7** Solve $x^2 - 10x + 25 = 0$.

**Answer:** $x = 5$

$(x - 5)(x - 5) = 0$

$x = 5$

$(-5) \times (-5) = 25$ *and* $(-5) + (-5) = -10$

*Both brackets give the same answer, so there is only one solution*

**Marks** M1 $(x - 5)(x - 5) = 0$ or $(x - 5)^2 = 0$ A1 $x = 5$ (only)

**8** Solve $x^2 + 9x = 0$.

**Answer:** $x = 0$ or $x = -9$

$x(x + 9) = 0$ *Common factor* $x$

$x = 0$ or $x = -9$

**Marks** M1 $x(x + 9) = 0$ A1 $x = 0$ and $x = -9$

**9** Solve $x^2 - 3x - 40 = 0$.

**Answer:** $x = 8$ or $x = -5$

$(x - 8)(x + 5) = 0$ $(-8) \times 5 = -40$ *and* $(-8) + 5 = -3$

$x = 8$ or $x = -5$

**Marks** M1 $(x \pm 8)(x \pm 5)$ A1 $(x - 8)(x + 5) = 0$ A1 $x = 8$ and $x = -5$

## Level 3 · Exam-style — 14 marks

Show that, hence and graph questions in real GCSE style.

**10** (a) Factorise $x^2 - 2x - 15$.

**Answer:** $(x - 5)(x + 3)$

$(-5) \times 3 = -15, \quad (-5) + 3 = -2$

$x^2 - 2x - 15 = (x - 5)(x + 3)$

**Marks** M1 $(x \pm 5)(x \pm 3)$ A1 $(x - 5)(x + 3)$

(b) Hence solve $x^2 - 2x - 15 = 0$.

**Answer:** $x = 5$ or $x = -3$

$(x - 5)(x + 3) = 0$ *Use part (a)*

$x = 5$ or $x = -3$

**Marks** B1 $x = 5$ and $x = -3$ — ft their (a)

**11** (a) A rectangle is $x$ cm wide and $(x+5)$ cm long. Its area is $36$ cm$^2$. Show that $x^2+5x-36=0$.

**Answer:** $x(x+5)=36$, so $x^2+5x=36$, so $x^2+5x-36=0$.

| | |
|---|---|
| $x(x+5)=36$ | *Area = width × length* |
| $x^2+5x=36$ | |
| $x^2+5x-36=0$ | *Subtract $36$ from both sides* |

**Marks** M1 $x(x+5)=36$ A1 Expands and rearranges correctly to $x^2+5x-36=0$

(b) Solve $x^2+5x-36=0$.

**Answer:** $x=4$ or $x=-9$

| | |
|---|---|
| $(x+9)(x-4)=0$ | $9\times(-4)=-36$ *and* $9+(-4)=5$ |
| $x=-9$ or $x=4$ | |

**Marks** M1 $(x\pm9)(x\pm4)$ A1 $(x+9)(x-4)=0$ A1 $x=4$ and $x=-9$

(c) Write down the width of the rectangle.

**Answer:** $4$ cm

| | |
|---|---|
| $x=4$ cm | *A width can't be negative, so reject $x=-9$* |

**Marks** B1 $4$ cm — ft a positive solution from (b)

**12** Ella solves $x^2-7x+10=0$. She writes $(x-2)(x-5)=0$, so $x=-2$ or $x=-5$. Explain Ella's mistake, and write down the correct solutions.

**Answer:** $x=2$ or $x=5$. Ella's brackets are right, but $x-2=0$ gives $x=2$, not $-2$.

| | |
|---|---|
| $(x-2)(x-5)=0$ | *Ella's factorising is correct* |
| $x-2=0 \Rightarrow x=2$ | *Add $2$ to both sides: the sign changes* |
| $x-5=0 \Rightarrow x=5$ | |

**Marks** B1 Explains that $x-2=0$ gives $x=2$ (the signs should change) B1 $x=2$ and $x=5$

**13** The graph of $y = x^2 - x - 12$ crosses the $x$-axis at two points. Use algebra to find the coordinates of these points.

**Answer:** $(4,\ 0)$ and $(-3,\ 0)$

| | |
|---|---|
| $x^2 - x - 12 = 0$ | *On the $x$-axis, $y = 0$* |
| $(x - 4)(x + 3) = 0$ | |
| $x = 4$ or $x = -3$ | |
| $(4,\ 0)$ and $(-3,\ 0)$ | *Give coordinates, not just the $x$ values* |

**Marks** M1 $x^2 - x - 12 = 0$ M1 $(x - 4)(x + 3) = 0$ A1 $(4,\ 0)$ and $(-3,\ 0)$

## Level 4 · Challenge (Foundation) — 11 marks

Grade 4–5 problem solving: forming and solving quadratic equations from words.

**14** (a) I think of a number, $x$. I square it, then subtract $3$ times the number. The answer is $28$. Show that $x^2 - 3x - 28 = 0$.

**Answer:** $x^2 - 3x = 28$, so $x^2 - 3x - 28 = 0$.

| | |
|---|---|
| $x^2 - 3x = 28$ | *Write the words as an equation* |
| $x^2 - 3x - 28 = 0$ | |

**Marks** B1 $x^2 - 3x = 28$ rearranged to $x^2 - 3x - 28 = 0$

(b) Find the two possible values of the number.

**Answer:** $7$ or $-4$

| | |
|---|---|
| $(x - 7)(x + 4) = 0$ | $(-7) \times 4 = -28$ *and* $(-7) + 4 = -3$ |
| $x = 7$ or $x = -4$ | *Check:* $7^2 - 3 \times 7 = 28$ *and* $(-4)^2 - 3 \times (-4) = 28$ |

**Marks** M1 $(x \pm 7)(x \pm 4)$ A1 $(x - 7)(x + 4) = 0$ A1 $7$ and $-4$

**15** (a) The product of two consecutive positive whole numbers is $132$. The smaller number is $x$. Show that $x^2 + x - 132 = 0$.

**Answer:** $x(x+1) = 132$, so $x^2 + x - 132 = 0$.

$x(x+1) = 132$ — *The next whole number is $x+1$*

$x^2 + x - 132 = 0$

**Marks** B1 $x(x+1) = 132$ expanded and rearranged to $x^2 + x - 132 = 0$

(b) Find the two numbers.

**Answer:** $11$ and $12$

$12 \times (-11) = -132, \quad 12 + (-11) = 1$ — *Look for factors of $132$ that differ by $1$*

$(x+12)(x-11) = 0$

$x = 11 \quad$ (or $x = -12$)

The numbers are $11$ and $12$ — *The numbers are positive, so reject $x = -12$*

**Marks** M1 $(x \pm 12)(x \pm 11)$ A1 $(x+12)(x-11) = 0$ A1 $11$ and $12$ (rejecting $x = -12$)

**16** A rectangle has length $(x+3)$ cm and width $(x-3)$ cm. Its area is $40$ cm$^2$. Work out the value of $x$.

**Answer:** $x = 7$

$(x+3)(x-3) = 40$ — *Area = length × width*

$x^2 - 9 = 40$ — *Difference of two squares: $(x+3)(x-3) = x^2 - 9$*

$x^2 = 49$

$x = 7$ — *$x = -7$ would make the width $-10$ cm, so reject it*

**Marks** M1 $(x+3)(x-3) = 40$ M1 $x^2 - 9 = 40$ or $x^2 = 49$ A1 $x = 7$ only