Solving Linear Equations: answers

Worked solutions and mark scheme · GCSE Maths · Higher tier

M = method mark, A = accuracy mark (needs the method), B = independent mark, ft = follow through from an earlier wrong answer.

Level 1 · Warm-up 11 marks

Two-step equations, brackets and unknowns on both sides.

1 Solve $3x + 5 = 20$.

Answer: $x = 5$

$3x = 15$	*Subtract* 5 *from both sides*
$x = 5$	*Divide both sides by* 3

Marks M1 $3x = 15$ A1 $x = 5$

2 Solve $\frac{x}{4} - 3 = 2$.

Answer: $x = 20$

$\frac{x}{4} = 5$	*Add* 3 *to both sides*
$x = 20$	*Multiply both sides by* 4

Marks M1 $\frac{x}{4} = 5$ A1 $x = 20$

3 Solve $5(x - 2) = 35$.

Answer: $x = 9$

$x - 2 = 7$	*Divide both sides by* 5 *(or expand to* $5x - 10 = 35$*)*
$x = 9$	

Marks M1 $5x - 10 = 35$ or $x - 2 = 7$ A1 $x = 9$

4 Solve $7x - 4 = 3x + 16$.

Answer: $x = 5$

$4x - 4 = 16$	*Subtract $3x$ from both sides*
$4x = 20$	*Add 4 to both sides*
$x = 5$	

Marks M1 Collects the x terms: $4x - 4 = 16$ or $7x - 3x = 16 + 4$ M1 $4x = 20$ A1 $x = 5$

5 Solve $2(3x + 1) = 5$.

Answer: $x = \frac{1}{2}$

$6x + 2 = 5$	*Expand the bracket*
$6x = 3$	
$x = \frac{3}{6} = \frac{1}{2}$	*An answer can be a fraction*

Marks M1 $6x + 2 = 5$ A1 $x = \frac{1}{2}$ oe

Level 2 · Build

17 marks

Negative answers, fractions, and equations with two fractions.

6 Solve $4(x + 3) = 2(x - 5)$.

Answer: $x = -11$

$4x + 12 = 2x - 10$	*Expand both brackets*
$2x + 12 = -10$	*Subtract $2x$ from both sides*
$2x = -22$	
$x = -11$	

Marks M1 $4x + 12 = 2x - 10$ M1 $2x = -22$ A1 $x = -11$

7 Solve $\frac{x+5}{3} = x - 3$.

Answer: $x = 7$

$x + 5 = 3(x - 3)$	*Multiply both sides by* 3
$x + 5 = 3x - 9$	
$14 = 2x$	*Subtract* x *and add* 9 *on both sides*
$x = 7$	

Marks **M1** $x + 5 = 3(x - 3)$ **M1** $x + 5 = 3x - 9$ and collects terms, e.g. $14 = 2x$ **A1** $x = 7$

8 Solve $12 - 3x = 2x - 8$.

Answer: $x = 4$

$12 = 5x - 8$	*Add* $3x$ *to both sides, so the* x *term stays positive*
$20 = 5x$	
$x = 4$	

Marks **M1** Collects the x terms on one side: e.g. $12 = 5x - 8$ **M1** $20 = 5x$ **A1** $x = 4$

9 Solve $\frac{2x-1}{5} = 3$.

Answer: $x = 8$

$2x - 1 = 15$	*Multiply both sides by* 5
$2x = 16$	
$x = 8$	

Marks **M1** $2x - 1 = 15$ **A1** $x = 8$

10 Solve $\frac{x+2}{4}+\frac{x-1}{5}=3$.

Answer: $x=6$

$5(x+2)+4(x-1)=60$ *Multiply every term, including the 3, by 20*

$5x+10+4x-4=60$

$9x+6=60$

$9x=54 \Rightarrow x=6$

Marks M1 Multiplies every term by 20: $5(x+2)+4(x-1)=60$ M1 $9x+6=60$ A1 $x=6$

11 Solve $\frac{3x-1}{2}-\frac{x+4}{3}=4$.

Answer: $x=5$

$3(3x-1)-2(x+4)=24$ *Multiply every term by 6*

$9x-3-2x-8=24$ *Careful: $-2(x+4)=-2x-8$*

$7x-11=24$

$7x=35 \Rightarrow x=5$

Marks M1 Multiplies every term by 6: $3(3x-1)-2(x+4)=24$ M1 $7x-11=24$ A1 $x=5$

Level 3 · Exam-style — 20 marks

Forming equations from angles, perimeters and ages, and spotting mistakes.

12 The angles of a triangle are $x°$, $(2x+10)°$ and $(3x-40)°$. Work out the size of the largest angle.

Answer: $80°$

$x+(2x+10)+(3x-40)=180$ *Angles in a triangle add up to 180°*

$6x-30=180$

$6x=210 \Rightarrow x=35$

$35°, \quad 2(35)+10=80°, \quad 3(35)-40=65°$ *Work out all three angles*

Largest angle $=80°$ *The question asks for the angle, not x*

Marks M1 $x+2x+10+3x-40=180$ M1 $6x-30=180$ A1 $x=35$ A1 $80°$

13 (a) A rectangle has length $(2x + 3)$ cm and width $(x - 1)$ cm. Its perimeter is 34 cm. Form an equation and solve it to find x.

Answer: $x = 5$

$2(2x + 3) + 2(x - 1) = 34$ — *Perimeter: two lengths and two widths*

$4x + 6 + 2x - 2 = 34$

$6x + 4 = 34$

$6x = 30 \Rightarrow x = 5$

Marks M1 $2(2x + 3) + 2(x - 1) = 34$ oe M1 $6x + 4 = 34$ A1 $x = 5$

(b) Work out the area of the rectangle.

Answer: 52 cm^2

length $= 2(5) + 3 = 13$, width $= 5 - 1 = 4$

area $= 13 \times 4 = 52$ cm^2

Marks M1 Length 13 and width 4 — ft their x A1 52 cm^2

14 Ali is 3 times as old as Ben. In 5 years' time, Ali will be twice as old as Ben. How old is Ben now?

Answer: 5 years old

Ben $= x$, Ali $= 3x$ — *Choose a letter for the unknown*

$3x + 5 = 2(x + 5)$ — *In 5 years both are 5 years older*

$3x + 5 = 2x + 10$

$x = 5$ — *Check: Ali is 15; in 5 years, $20 = 2 \times 10$*

Marks M1 $3x + 5 = 2(x + 5)$, where Ben is x now M1 $3x + 5 = 2x + 10$ A1 5

15 Leah solves $4x - 7 = 2x + 9$. Her working is: $4x - 2x = 9 - 7$, so $2x = 2$, so $x = 1$. Explain Leah's mistake, and solve the equation correctly.

Answer: Moving -7 to the other side makes it $+7$, not -7. $2x = 16$, so $x = 8$.

$4x - 2x = 9 + 7$ — *Adding 7 to both sides gives $+7$ on the right*

$2x = 16$

$x = 8$

Marks B1 Explains that the -7 becomes $+7$ on the other side (she should add 7) B1 $x = 8$

16 Solve $\frac{3}{x-2} = \frac{5}{x+1}$.

Answer: $x = 6.5$

$3(x+1) = 5(x-2)$ — *Multiply both sides by $(x-2)(x+1)$: cross-multiply*

$3x + 3 = 5x - 10$

$13 = 2x$

$x = 6.5$

Marks M1 $3(x+1) = 5(x-2)$ M1 $3x + 3 = 5x - 10$ A1 $x = 6.5$ oe

17 Solve $(x+1)^2 = (x-3)^2 + 16$.

Answer: $x = 3$

$x^2 + 2x + 1 = x^2 - 6x + 9 + 16$ — *Expand both brackets*

$2x + 1 = -6x + 25$ — *The x^2 terms cancel, so it is linear*

$8x = 24$

$x = 3$

Marks M1 Expands both squares: $x^2 + 2x + 1$ and $x^2 - 6x + 9$ M1 $8x = 24$ or $2x + 1 = -6x + 25$ A1 $x = 3$

Level 4 · Challenge — 12 marks

Grade 8–9: unknown constants, no solutions, and fractions with x on the bottom.

18 Show that the equation $2(3x+1) - 3(2x-1) = 4$ has no solution.

Answer: Expanding gives $6x + 2 - 6x + 3 = 4$, so $5 = 4$, which is false. No value of x works.

$6x + 2 - 6x + 3 = 4$ — *Careful: $-3(2x-1) = -6x + 3$*

$5 = 4$ — *The x terms cancel and leave a false statement*

No value of x makes it true

Marks M1 Expands correctly: $6x + 2 - 6x + 3$ A1 $5 = 4$ (or $5 \neq 4$) with a conclusion that there is no solution

19 Solve $\frac{x+3}{x-2} = \frac{x+1}{x-1}$.

Answer: $x = \frac{1}{3}$

$(x+3)(x-1) = (x+1)(x-2)$	*Cross-multiply*
$x^2 + 2x - 3 = x^2 - x - 2$	*Expand both sides*
$2x - 3 = -x - 2$	*The x^2 terms cancel*
$3x = 1 \Rightarrow x = \frac{1}{3}$	

Marks **M1** $(x+3)(x-1) = (x+1)(x-2)$ **M1** $x^2 + 2x - 3 = x^2 - x - 2$ **M1** $3x = 1$ **A1** $x = \frac{1}{3}$

20 The equation $\frac{2x+k}{3} = x - k$ has solution $x = 7$. Find the value of the constant k.

Answer: $k = \frac{7}{4}$

$\frac{14+k}{3} = 7 - k$	*Put $x = 7$ into the equation: now k is the unknown*
$14 + k = 21 - 3k$	*Multiply both sides by 3*
$4k = 7$	
$k = \frac{7}{4}$	

Marks **M1** Substitutes $x = 7$: $\frac{14+k}{3} = 7 - k$ **M1** $14 + k = 21 - 3k$ **A1** $k = \frac{7}{4}$ oe

21 Jo cycles to work at 15 km/h and walks home along the same route at 5 km/h. The two journeys take 2 hours altogether. How far is it from Jo's home to work?

Answer: 7.5 km

$\frac{d}{15} + \frac{d}{5} = 2$	*Time = distance ÷ speed, for each journey*
$d + 3d = 30$	*Multiply every term by 15*
$4d = 30 \Rightarrow d = 7.5$	

Marks **M1** $\frac{d}{15} + \frac{d}{5} = 2$ **M1** $d + 3d = 30$ or $\frac{4d}{15} = 2$ **A1** 7.5 km