Simultaneous Equations: answers

Worked solutions and mark scheme · GCSE Maths · Higher tier

M = method mark, A = accuracy mark (needs the method), B = independent mark, ft = follow through from an earlier wrong answer.

Level 1 · Warm-up 15 marks

Adding or subtracting the equations straight away, and simple substitution.

1 Solve the simultaneous equations $x + y = 10$ and $x - y = 4$.

Answer: $x = 7,\ y = 3$

$2x = 14$	*Add: $+y$ and $-y$ cancel*
$x = 7$	
$7 + y = 10 \Rightarrow y = 3$	*Substitute into the first equation*

Marks M1 Adds the equations: $2x = 14$ A1 $x = 7$ A1 $y = 3$

2 Solve the simultaneous equations $2x + y = 11$ and $x + y = 7$.

Answer: $x = 4,\ y = 3$

$x = 4$	*Subtract: both have $+y$, so the y terms cancel*
$4 + y = 7 \Rightarrow y = 3$	

Marks M1 Subtracts the equations: $x = 4$ A1 $x = 4$ A1 $y = 3$

3 Solve the simultaneous equations $3x + 2y = 16$ and $x - 2y = 0$.

Answer: $x = 4,\ y = 2$

$4x = 16$	*Add: $+2y$ and $-2y$ cancel*
$x = 4$	
$4 - 2y = 0 \Rightarrow y = 2$	

Marks M1 Adds the equations: $4x = 16$ A1 $x = 4$ A1 $y = 2$

4 Solve the simultaneous equations $2x + 3y = 13$ and $2x + y = 7$.

Answer: $x = 2,\ y = 3$

$2y = 6$ *Subtract: both have $2x$*

$y = 3$

$2x + 3 = 7 \Rightarrow x = 2$

Marks M1 Subtracts the equations: $2y = 6$ A1 $y = 3$ A1 $x = 2$

5 Solve the simultaneous equations $y = 2x$ and $x + y = 15$.

Answer: $x = 5,\ y = 10$

$x + 2x = 15$ *Replace y with $2x$*

$3x = 15 \Rightarrow x = 5$

$y = 2 \times 5 = 10$

Marks M1 Substitutes: $x + 2x = 15$ A1 $x = 5$ A1 $y = 10$

Level 2 · Build

24 marks

Multiplying one or both equations first, and a first look at a linear and a quadratic.

6 Solve the simultaneous equations $3x + 2y = 19$ and $x + 4y = 13$.

Answer: $x = 5,\ y = 2$

$6x + 4y = 38$ *Multiply the first equation by 2 to match the $4y$*

$5x = 25$ *Subtract $x + 4y = 13$*

$x = 5$

$5 + 4y = 13 \Rightarrow y = 2$

Marks M1 Multiplies the first by 2 and subtracts: $5x = 25$, or equivalent A1 $x = 5$ A1 $y = 2$

7 Solve the simultaneous equations $5x - 2y = 11$ and $3x + 4y = 4$.

Answer: $x = 2,\ y = -\frac{1}{2}$

$10x - 4y = 22$	*Multiply the first equation by* 2
$13x = 26$	*Add:* $-4y$ *and* $+4y$ *cancel*
$x = 2$	
$3(2) + 4y = 4 \Rightarrow 4y = -2 \Rightarrow y = -\frac{1}{2}$	

Marks M1 Multiplies the first by 2: $10x - 4y = 22$ M1 Adds: $13x = 26$ A1 $x = 2$ A1 $y = -\frac{1}{2}$ oe

8 Solve the simultaneous equations $4x + 3y = 2$ and $3x + 2y = 1$.

Answer: $x = -1,\ y = 2$

$8x + 6y = 4$	*Multiply the first equation by* 2
$9x + 6y = 3$	*Multiply the second equation by* 3
$x = -1$	*Subtract the first from the second*
$4(-1) + 3y = 2 \Rightarrow 3y = 6 \Rightarrow y = 2$	

Marks M1 Multiplies both to match a coefficient, e.g. $8x + 6y = 4$ and $9x + 6y = 3$ M1 Subtracts correctly: $x = -1$ A1 $x = -1$ A1 $y = 2$

9 Solve the simultaneous equations $y = 3x - 5$ and $y = x + 7$.

Answer: $x = 6,\ y = 13$

$3x - 5 = x + 7$	*Both equal* y*, so they equal each other*
$2x = 12 \Rightarrow x = 6$	
$y = 6 + 7 = 13$	

Marks M1 $3x - 5 = x + 7$ A1 $x = 6$ A1 $y = 13$

10 Solve the simultaneous equations $y = x^2$ and $y = x + 6$.

Answer: $x = 3,\ y = 9$ and $x = -2,\ y = 4$

$x^2 = x + 6$ — *Substitute $y = x + 6$ into $y = x^2$*

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0 \Rightarrow x = 3$ or $x = -2$

$x = 3 : y = 9; \quad x = -2 : y = 4$ — *Use the linear equation to find each y*

Marks M1 $x^2 = x + 6$ A1 $x^2 - x - 6 = 0$ M1 $(x - 3)(x + 2) = 0$ A1 $x = 3$ and $x = -2$ A1 $y = 9$ and $y = 4$, correctly paired

11 Solve the simultaneous equations $y = x^2 - 3x$ and $y = 2x - 4$.

Answer: $x = 1,\ y = -2$ and $x = 4,\ y = 4$

$x^2 - 3x = 2x - 4$

$x^2 - 5x + 4 = 0$

$(x - 1)(x - 4) = 0 \Rightarrow x = 1$ or $x = 4$

$x = 1 : y = -2; \quad x = 4 : y = 4$

Marks M1 $x^2 - 3x = 2x - 4$ A1 $x^2 - 5x + 4 = 0$ M1 $(x - 1)(x - 4) = 0$ A1 $x = 1$ and $x = 4$ A1 $y = -2$ and $y = 4$, correctly paired

Level 3 · Exam-style 24 marks
Forming equations from prices, numbers and animals, and spotting mistakes.

12 4 coffees and 3 cakes cost £14.50. 2 coffees and 5 cakes cost £12.50. Work out the cost of one coffee and the cost of one cake.

Answer: Coffee £2.50, cake £1.50

$4c + 3k = 1450, \quad 2c + 5k = 1250$ — *Work in pence to avoid decimals*

$4c + 10k = 2500$ — *Multiply the second equation by* 2

$7k = 1050 \Rightarrow k = 150$ — *Subtract the first equation*

$4c + 450 = 1450 \Rightarrow c = 250$

coffee £2.50, cake £1.50

Marks M1 $4c + 3k = 1450$ and $2c + 5k = 1250$ (in pence) oe M1 Correct method to eliminate one letter, e.g. $7k = 1050$ A1 Cake £1.50 A1 Coffee £2.50

13 The sum of two numbers is 42 and their difference is 8. Work out the two numbers.

Answer: 25 and 17

$x + y = 42, \quad x - y = 8$

$2x = 50 \Rightarrow x = 25$ — *Add the equations*

$y = 42 - 25 = 17$

Marks M1 $x + y = 42$ and $x - y = 8$ M1 $2x = 50$ A1 25 and 17

14 Mia solves $x + 2y = 11$ and $x + y = 7$. She subtracts the equations and writes $y = 18$. Explain Mia's mistake, and solve the equations correctly.

Answer: She added the right-hand sides ($11 + 7$) instead of subtracting them. $y = 4$, $x = 3$.

$(x + 2y) - (x + y) = 11 - 7$ — *Subtract both sides: the numbers on the right are subtracted too*

$y = 4$

$x + 4 = 7 \Rightarrow x = 3$

Marks B1 Explains that $11 - 7 = 4$ should be used, not $11 + 7$ A1 $y = 4$ A1 $x = 3$

15 A farmer has some chickens and some sheep. Altogether there are 30 heads and 80 legs. How many chickens and how many sheep are there?

Answer: 20 chickens and 10 sheep

$c + s = 30$	*Each animal has one head*
$2c + 4s = 80$	*Chickens have 2 legs, sheep have 4*
$2c + 2s = 60$	*Multiply the first equation by 2*
$2s = 20 \Rightarrow s = 10, \quad c = 20$	*Subtract*

Marks **M1** $c + s = 30$ **M1** $2c + 4s = 80$ **A1** $s = 10$ **A1** $c = 20$

16 Solve the simultaneous equations $x^2 + y^2 = 25$ and $y = x + 1$.

Answer: $x = 3,\ y = 4$ and $x = -4,\ y = -3$

$x^2 + (x+1)^2 = 25$	*Substitute the linear equation into the circle*
$x^2 + x^2 + 2x + 1 = 25$	
$2x^2 + 2x - 24 = 0 \Rightarrow x^2 + x - 12 = 0$	
$(x+4)(x-3) = 0 \Rightarrow x = 3$ or $x = -4$	
$x = 3 : y = 4; \quad x = -4 : y = -3$	

Marks **M1** $x^2 + (x+1)^2 = 25$ **A1** $2x^2 + 2x - 24 = 0$ or $x^2 + x - 12 = 0$ **M1** $(x+4)(x-3) = 0$ **A1** $x = 3$ and $x = -4$ **A1** $y = 4$ and $y = -3$, correctly paired

17 Solve the simultaneous equations $y = 2x^2 - 5x + 1$ and $y = x - 3$.

Answer: $x = 1,\ y = -2$ and $x = 2,\ y = -1$

$2x^2 - 5x + 1 = x - 3$	
$2x^2 - 6x + 4 = 0 \Rightarrow x^2 - 3x + 2 = 0$	*Divide by 2*
$(x-1)(x-2) = 0 \Rightarrow x = 1$ or $x = 2$	
$x = 1 : y = -2; \quad x = 2 : y = -1$	

Marks **M1** $2x^2 - 5x + 1 = x - 3$ **A1** $2x^2 - 6x + 4 = 0$ or $x^2 - 3x + 2 = 0$ **M1** $(x-1)(x-2) = 0$ **A1** $x = 1$ and $x = 2$ **A1** $y = -2$ and $y = -1$, correctly paired

Level 4 · Challenge 20 marks
Grade 8–9: circles, decimal answers, no solutions and a product of two unknowns.

18 Solve the simultaneous equations $x^2 + y^2 = 20$ and $y = 2x + 1$. Give your answers correct to 2 decimal places.

Answer: $x = 1.59,\ y = 4.18$ and $x = -2.39,\ y = -3.78$

$x^2 + (2x+1)^2 = 20$

$x^2 + 4x^2 + 4x + 1 = 20 \Rightarrow 5x^2 + 4x - 19 = 0$

$x = \dfrac{-4 \pm \sqrt{16 + 380}}{10} = \dfrac{-4 \pm \sqrt{396}}{10}$ — *It doesn't factorise, so use the formula*

$x = 1.589\ldots$ or $x = -2.389\ldots$

$y = 2(1.589\ldots) + 1 = 4.179\ldots$ — *Use the unrounded x values*

$y = 2(-2.389\ldots) + 1 = -3.779\ldots$

$(1.59,\ 4.18)$ and $(-2.39,\ -3.78)$

Marks M1 $x^2 + (2x+1)^2 = 20$ A1 $5x^2 + 4x - 19 = 0$ M1 Correct substitution into the quadratic formula A1 $x = 1.59$ and $x = -2.39$ M1 Substitutes their x values into $y = 2x+1$ A1 $y = 4.18$ and $y = -3.78$, correctly paired

19 Show that the line $y = x + 3$ and the curve $y = x^2 + 5$ do not meet.

Answer: $x^2 + 5 = x + 3$ gives $x^2 - x + 2 = 0$, and $x^2 - x + 2 = \left(x - \frac{1}{2}\right)^2 + \frac{7}{4}$, which is always at least $\frac{7}{4}$, so it is never 0.

$x^2 + 5 = x + 3 \Rightarrow x^2 - x + 2 = 0$ — *Where they meet, the y values are equal*

$x^2 - x + 2 = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} + 2 = \left(x - \frac{1}{2}\right)^2 + \frac{7}{4}$ — *Complete the square*

$\left(x - \frac{1}{2}\right)^2 + \frac{7}{4} \geq \frac{7}{4} > 0$ — *A square is never negative*

No solutions, so they do not meet

Marks M1 $x^2 - x + 2 = 0$ M1 $\left(x - \frac{1}{2}\right)^2 + \frac{7}{4}$, or the formula with a negative under the square root A1 Conclusion: it can never equal 0, so the line and curve do not meet

20 Solve the simultaneous equations $2x + 3y = 7$ and $xy = 2$.

Answer: $x = 2,\ y = 1$ and $x = \frac{3}{2},\ y = \frac{4}{3}$

$y = \dfrac{7 - 2x}{3}$ — *Rearrange the linear equation*

$x \cdot \dfrac{7 - 2x}{3} = 2 \ \Rightarrow\ 7x - 2x^2 = 6$

$2x^2 - 7x + 6 = 0$

$(2x - 3)(x - 2) = 0 \ \Rightarrow\ x = \frac{3}{2}$ or $x = 2$

$x = 2 : y = 1; \quad x = \frac{3}{2} : y = \frac{4}{3}$ — *Use $xy = 2$, so $y = \frac{2}{x}$*

Marks M1 Rearranges the linear equation, e.g. $y = \dfrac{7 - 2x}{3}$ M1 Substitutes: $x \cdot \dfrac{7 - 2x}{3} = 2$ A1 $2x^2 - 7x + 6 = 0$ M1 $(2x - 3)(x - 2) = 0$ A1 $x = 2$ and $x = \frac{3}{2}$ A1 $y = 1$ and $y = \frac{4}{3}$, correctly paired

21 A rectangle has a perimeter of 26 cm and an area of 40 cm^2. Work out its length and width.

Answer: Length 8 cm, width 5 cm

$l + w = 13, \quad lw = 40$ — *Half the perimeter is 13*

$w(13 - w) = 40$ — *Substitute $l = 13 - w$*

$13w - w^2 = 40 \ \Rightarrow\ w^2 - 13w + 40 = 0$

$(w - 5)(w - 8) = 0 \ \Rightarrow\ w = 5$ or $w = 8$

The sides are 8 cm and 5 cm — *Both solutions give the same rectangle*

Marks M1 $2l + 2w = 26$ (or $l + w = 13$) and $lw = 40$ M1 Substitutes: $w(13 - w) = 40$ A1 $w^2 - 13w + 40 = 0$ M1 $(w - 5)(w - 8) = 0$ A1 8 cm by 5 cm