Simultaneous Equations: answers

Worked solutions and mark scheme · GCSE Maths · Foundation tier

M = method mark, A = accuracy mark (needs the method), B = independent mark, ft = follow through from an earlier wrong answer.

Level 1 · Warm-up 15 marks

Adding or subtracting the equations straight away, and simple substitution.

1 Solve the simultaneous equations $x + y = 10$ and $x - y = 4$.

Answer: $x = 7,\ y = 3$

$2x = 14$	*Add: $+y$ and $-y$ cancel*
$x = 7$	
$7 + y = 10 \Rightarrow y = 3$	*Substitute into the first equation*

Marks M1 Adds the equations: $2x = 14$ A1 $x = 7$ A1 $y = 3$

2 Solve the simultaneous equations $2x + y = 11$ and $x + y = 7$.

Answer: $x = 4,\ y = 3$

$x = 4$	*Subtract: both have $+y$, so the y terms cancel*
$4 + y = 7 \Rightarrow y = 3$	

Marks M1 Subtracts the equations: $x = 4$ A1 $x = 4$ A1 $y = 3$

3 Solve the simultaneous equations $3x + 2y = 16$ and $x - 2y = 0$.

Answer: $x = 4,\ y = 2$

$4x = 16$	*Add: $+2y$ and $-2y$ cancel*
$x = 4$	
$4 - 2y = 0 \Rightarrow y = 2$	

Marks M1 Adds the equations: $4x = 16$ A1 $x = 4$ A1 $y = 2$

4 Solve the simultaneous equations $2x + 3y = 13$ and $2x + y = 7$.

Answer: $x = 2,\ y = 3$

$2y = 6$ *Subtract: both have $2x$*

$y = 3$

$2x + 3 = 7 \Rightarrow x = 2$

Marks M1 Subtracts the equations: $2y = 6$ A1 $y = 3$ A1 $x = 2$

5 Solve the simultaneous equations $y = 2x$ and $x + y = 15$.

Answer: $x = 5,\ y = 10$

$x + 2x = 15$ *Replace y with $2x$*

$3x = 15 \Rightarrow x = 5$

$y = 2 \times 5 = 10$

Marks M1 Substitutes: $x + 2x = 15$ A1 $x = 5$ A1 $y = 10$

Level 2 · Build

14 marks

Multiplying one or both equations first, and a first look at a linear and a quadratic.

6 Solve the simultaneous equations $3x + 2y = 19$ and $x + 4y = 13$.

Answer: $x = 5,\ y = 2$

$6x + 4y = 38$ *Multiply the first equation by 2 to match the $4y$*

$5x = 25$ *Subtract $x + 4y = 13$*

$x = 5$

$5 + 4y = 13 \Rightarrow y = 2$

Marks M1 Multiplies the first by 2 and subtracts: $5x = 25$, or equivalent A1 $x = 5$ A1 $y = 2$

7 Solve the simultaneous equations $5x - 2y = 11$ and $3x + 4y = 4$.

Answer: $x = 2,\ y = -\frac{1}{2}$

$10x - 4y = 22$	*Multiply the first equation by* 2
$13x = 26$	*Add:* $-4y$ *and* $+4y$ *cancel*
$x = 2$	
$3(2) + 4y = 4 \Rightarrow 4y = -2 \Rightarrow y = -\frac{1}{2}$	

Marks M1 Multiplies the first by 2: $10x - 4y = 22$ M1 Adds: $13x = 26$ A1 $x = 2$ A1 $y = -\frac{1}{2}$ oe

8 Solve the simultaneous equations $4x + 3y = 2$ and $3x + 2y = 1$.

Answer: $x = -1,\ y = 2$

$8x + 6y = 4$	*Multiply the first equation by* 2
$9x + 6y = 3$	*Multiply the second equation by* 3
$x = -1$	*Subtract the first from the second*
$4(-1) + 3y = 2 \Rightarrow 3y = 6 \Rightarrow y = 2$	

Marks M1 Multiplies both to match a coefficient, e.g. $8x + 6y = 4$ and $9x + 6y = 3$ M1 Subtracts correctly: $x = -1$ A1 $x = -1$ A1 $y = 2$

9 Solve the simultaneous equations $y = 3x - 5$ and $y = x + 7$.

Answer: $x = 6,\ y = 13$

$3x - 5 = x + 7$	*Both equal* y*, so they equal each other*
$2x = 12 \Rightarrow x = 6$	
$y = 6 + 7 = 13$	

Marks M1 $3x - 5 = x + 7$ A1 $x = 6$ A1 $y = 13$

Level 3 · Exam-style 14 marks
Forming equations from prices, numbers and animals, and spotting mistakes.

10 4 coffees and 3 cakes cost £14.50. 2 coffees and 5 cakes cost £12.50. Work out the cost of one coffee and the cost of one cake.

Answer: Coffee £2.50, cake £1.50

$4c + 3k = 1450, \quad 2c + 5k = 1250$ — *Work in pence to avoid decimals*

$4c + 10k = 2500$ — *Multiply the second equation by* 2

$7k = 1050 \Rightarrow k = 150$ — *Subtract the first equation*

$4c + 450 = 1450 \Rightarrow c = 250$

coffee £2.50, cake £1.50

Marks M1 $4c + 3k = 1450$ and $2c + 5k = 1250$ (in pence) oe M1 Correct method to eliminate one letter, e.g. $7k = 1050$ A1 Cake £1.50 A1 Coffee £2.50

11 The sum of two numbers is 42 and their difference is 8. Work out the two numbers.

Answer: 25 and 17

$x + y = 42, \quad x - y = 8$

$2x = 50 \Rightarrow x = 25$ — *Add the equations*

$y = 42 - 25 = 17$

Marks M1 $x + y = 42$ and $x - y = 8$ M1 $2x = 50$ A1 25 and 17

12 Mia solves $x + 2y = 11$ and $x + y = 7$. She subtracts the equations and writes $y = 18$. Explain Mia's mistake, and solve the equations correctly.

Answer: She added the right-hand sides ($11 + 7$) instead of subtracting them. $y = 4$, $x = 3$.

$(x + 2y) - (x + y) = 11 - 7$ — *Subtract both sides: the numbers on the right are subtracted too*

$y = 4$

$x + 4 = 7 \Rightarrow x = 3$

Marks B1 Explains that $11 - 7 = 4$ should be used, not $11 + 7$ A1 $y = 4$ A1 $x = 3$

13 A farmer has some chickens and some sheep. Altogether there are 30 heads and 80 legs. How many chickens and how many sheep are there?

Answer: 20 chickens and 10 sheep

$c + s = 30$	*Each animal has one head*
$2c + 4s = 80$	*Chickens have 2 legs, sheep have 4*
$2c + 2s = 60$	*Multiply the first equation by 2*
$2s = 20 \Rightarrow s = 10, \quad c = 20$	*Subtract*

Marks M1 $c + s = 30$ M1 $2c + 4s = 80$ A1 $s = 10$ A1 $c = 20$

Level 4 · Challenge (Foundation) — 11 marks

Grade 4–5 problem solving: forming and solving a pair of equations from words.

14 Adult tickets for a show cost £8 and child tickets cost £5. 40 tickets were sold for £260 altogether. How many adult tickets were sold?

Answer: 20 adult tickets

$a + c = 40, \quad 8a + 5c = 260$	
$5a + 5c = 200$	*Multiply the first equation by 5*
$3a = 60 \Rightarrow a = 20$	*Subtract*

Marks M1 $a + c = 40$ M1 $8a + 5c = 260$ M1 Correct elimination, e.g. $3a = 60$ A1 20

15 The straight line $y = mx + c$ passes through the points $(1,\ 5)$ and $(3,\ 11)$. Find the values of m and c.

Answer: $m = 3,\ c = 2$

$5 = m + c, \quad 11 = 3m + c$	*Substitute each point into $y = mx + c$*
$6 = 2m \Rightarrow m = 3$	*Subtract the first from the second*
$c = 5 - 3 = 2$	

Marks M1 $5 = m + c$ and $11 = 3m + c$ A1 $m = 3$ A1 $c = 2$

16 In 2 years' time, Jo will be twice as old as Sam. Two years ago, Jo was 3 times as old as Sam. How old are Jo and Sam now?

Answer: Jo is 14, Sam is 6

$$j + 2 = 2(s + 2) \Rightarrow j = 2s + 2$$
$$j - 2 = 3(s - 2) \Rightarrow j = 3s - 4$$
$$2s + 2 = 3s - 4 \Rightarrow s = 6$$
$$j = 2(6) + 2 = 14$$

Check: in 2 years, $16 = 2 \times 8$; two years ago, $12 = 3 \times 4$

Marks M1 $j + 2 = 2(s + 2)$ M1 $j - 2 = 3(s - 2)$ M1 Simplifies to $j = 2s + 2$ and $j = 3s - 4$, and solves A1 Jo 14 and Sam 6