Rearranging Formulae: answers

Worked solutions and mark scheme · GCSE Maths · Higher tier

M = method mark, A = accuracy mark (needs the method), B = independent mark, ft = follow through from an earlier wrong answer.

Level 1 · Warm-up 9 marks

One or two steps: adding, subtracting, multiplying and dividing.

1 Make t the subject of $v = u + at$.

Answer: $t = \dfrac{v - u}{a}$

$v - u = at$	*Subtract u from both sides*
$t = \dfrac{v - u}{a}$	*Divide both sides by a*

Marks M1 $v - u = at$ A1 $t = \dfrac{v - u}{a}$

2 Make x the subject of $y = 4x - 3$.

Answer: $x = \dfrac{y + 3}{4}$

$y + 3 = 4x$	*Add 3 to both sides*
$x = \dfrac{y + 3}{4}$	*Divide both sides by 4*

Marks M1 $y + 3 = 4x$ A1 $x = \dfrac{y + 3}{4}$

3 Make r the subject of $C = 2\pi r$.

Answer: $r = \dfrac{C}{2\pi}$

$r = \dfrac{C}{2\pi}$	*Divide both sides by 2π*

Marks B1 $r = \dfrac{C}{2\pi}$ oe

4 Make h the subject of $A = \frac{1}{2}bh$.

Answer: $h = \dfrac{2A}{b}$

$2A = bh$	*Multiply both sides by* 2
$h = \dfrac{2A}{b}$	*Divide both sides by* b

Marks **M1** $2A = bh$ **A1** $h = \dfrac{2A}{b}$

5 Make m the subject of $P = 3m + 2n$.

Answer: $m = \dfrac{P - 2n}{3}$

$P - 2n = 3m$	*Subtract* $2n$ *from both sides*
$m = \dfrac{P - 2n}{3}$	

Marks **M1** $P - 2n = 3m$ **A1** $m = \dfrac{P - 2n}{3}$

Level 2 · Build

13 marks

Fractions and negatives, and a first look at squares and roots.

6 Make x the subject of $y = \dfrac{x + 5}{2}$.

Answer: $x = 2y - 5$

$2y = x + 5$	*Multiply both sides by* 2 *first*
$x = 2y - 5$	

Marks **M1** $2y = x + 5$ **A1** $x = 2y - 5$

7 Make v the subject of $s = \frac{1}{2}(u + v)t$.

Answer: $v = \dfrac{2s}{t} - u$

$2s = (u + v)t$	*Multiply both sides by* 2
$\dfrac{2s}{t} = u + v$	*Divide both sides by* t
$v = \dfrac{2s}{t} - u$	

Marks **M1** $2s = (u + v)t$ **M1** $\dfrac{2s}{t} = u + v$ **A1** $v = \dfrac{2s}{t} - u$ oe, e.g. $\dfrac{2s - ut}{t}$

8 Make x the subject of $y = 7 - 2x$.

Answer: $x = \dfrac{7 - y}{2}$

$y + 2x = 7$	*Add* $2x$ *to both sides, so the* x *term is positive*
$2x = 7 - y$	
$x = \dfrac{7 - y}{2}$	

Marks **M1** $2x = 7 - y$ or $-2x = y - 7$ **A1** $x = \dfrac{7 - y}{2}$ oe

9 Make c the subject of $a = b - 3c$.

Answer: $c = \dfrac{b - a}{3}$

$a + 3c = b$	*Add* $3c$ *to both sides*
$3c = b - a$	
$c = \dfrac{b - a}{3}$	

Marks **M1** $3c = b - a$ **A1** $c = \dfrac{b - a}{3}$ oe

10 Make r the subject of $A = \pi r^2$.

Answer: $r = \sqrt{\frac{A}{\pi}}$

$r^2 = \frac{A}{\pi}$	*Divide both sides by* π
$r = \sqrt{\frac{A}{\pi}}$	*Square root both sides (a radius is positive)*

Marks M1 $r^2 = \frac{A}{\pi}$ A1 $r = \sqrt{\frac{A}{\pi}}$

11 Make x the subject of $y = \sqrt{x + 4}$.

Answer: $x = y^2 - 4$

$y^2 = x + 4$	*Square both sides to undo the square root*
$x = y^2 - 4$	

Marks M1 $y^2 = x + 4$ A1 $x = y^2 - 4$

Level 3 · Exam-style — 16 marks

Formulae in context, using your rearranged formula, and spotting mistakes.

12 (a) The formula $F = \frac{9C}{5} + 32$ converts a temperature in degrees Celsius, C, to degrees Fahrenheit, F. Make C the subject of the formula.

Answer: $C = \frac{5(F - 32)}{9}$

$F - 32 = \frac{9C}{5}$	*Subtract* 32 *from both sides*
$5(F - 32) = 9C$	*Multiply both sides by* 5
$C = \frac{5(F - 32)}{9}$	*Divide both sides by* 9

Marks M1 $F - 32 = \frac{9C}{5}$ or $5F = 9C + 160$ A1 $C = \frac{5(F - 32)}{9}$ oe

(b) Use your formula to convert $77°$F to degrees Celsius.

Answer: $25°$C

$$C = \frac{5(77 - 32)}{9} = \frac{5 \times 45}{9} = 25$$

Marks B1 25 — ft their formula

13 (a) The cost, C pounds, of hiring a van for d days is $C = 45 + 30d$. Make d the subject of the formula.

Answer: $d = \dfrac{C - 45}{30}$

$$C - 45 = 30d$$
$$d = \frac{C - 45}{30}$$

Marks M1 $C - 45 = 30d$ A1 $d = \dfrac{C - 45}{30}$

(b) Hal paid £195 to hire a van. For how many days did he hire it?

Answer: 5 days

$$d = \frac{195 - 45}{30} = \frac{150}{30} = 5$$

Marks B1 5 — ft their formula

14 Ravi makes x the subject of $y = 3x + 5$ and writes $x = \dfrac{y}{3} - 5$. Explain Ravi's mistake, and write down the correct answer.

Answer: He divided only y by 3; the whole of $y - 5$ must be divided by 3. $x = \dfrac{y - 5}{3}$.

$$y - 5 = 3x$$ *Subtract 5 first*
$$x = \frac{y - 5}{3}$$ *Then divide the whole of $y - 5$ by 3*

Marks B1 Explains that 5 must be subtracted before dividing, or that all of $y - 5$ is divided by 3 B1 $x = \dfrac{y - 5}{3}$

15 The kinetic energy of a moving object is $E = \frac{1}{2}mv^2$. Make m the subject of the formula.

Answer: $m = \dfrac{2E}{v^2}$

$2E = mv^2$	*Multiply both sides by* 2
$m = \dfrac{2E}{v^2}$	*Divide both sides by* v^2

Marks M1 $2E = mv^2$ A1 $m = \dfrac{2E}{v^2}$

16 Make u the subject of $v^2 = u^2 + 2as$, where u is positive.

Answer: $u = \sqrt{v^2 - 2as}$

$u^2 = v^2 - 2as$	*Subtract* $2as$ *from both sides*
$u = \sqrt{v^2 - 2as}$	*Square root the whole side: it is not* $v - \sqrt{2as}$

Marks M1 $u^2 = v^2 - 2as$ M1 Square roots the whole of the right-hand side A1 $u = \sqrt{v^2 - 2as}$

17 Make x the subject of $ax + b = cx + d$.

Answer: $x = \dfrac{d - b}{a - c}$

$ax - cx = d - b$	*Get both* x *terms on the same side*
$x(a - c) = d - b$	*Factorise out* x
$x = \dfrac{d - b}{a - c}$	*Divide by the bracket*

Marks M1 Collects the x terms: $ax - cx = d - b$ M1 $x(a - c) = d - b$ A1 $x = \dfrac{d - b}{a - c}$ oe

Level 4 · Challenge 13 marks
Grade 8–9: the subject in two places, fractions with the subject on the bottom, and roots.

18 Make x the subject of $y = \dfrac{x+2}{x-3}$.

Answer: $x = \dfrac{3y+2}{y-1}$

$y(x-3) = x+2$	*Multiply both sides by* $(x-3)$
$xy - 3y = x+2$	
$xy - x = 3y+2$	*Get both* x *terms on one side*
$x(y-1) = 3y+2$	*Factorise out* x
$x = \dfrac{3y+2}{y-1}$	

Marks M1 $y(x-3) = x+2$ M1 $xy - x = 3y+2$ M1 $x(y-1) = 3y+2$ A1 $x = \dfrac{3y+2}{y-1}$ oe

19 The time for one swing of a pendulum is $T = 2\pi\sqrt{\dfrac{l}{g}}$. Make l the subject of the formula.

Answer: $l = \dfrac{gT^2}{4\pi^2}$

$\dfrac{T}{2\pi} = \sqrt{\dfrac{l}{g}}$	*Divide both sides by* 2π
$\dfrac{T^2}{4\pi^2} = \dfrac{l}{g}$	*Square both sides:* $(2\pi)^2 = 4\pi^2$
$l = \dfrac{gT^2}{4\pi^2}$	*Multiply both sides by* g

Marks M1 $\dfrac{T}{2\pi} = \sqrt{\dfrac{l}{g}}$ M1 $\dfrac{T^2}{4\pi^2} = \dfrac{l}{g}$ A1 $l = \dfrac{gT^2}{4\pi^2}$

20 Make x the subject of $\dfrac{x}{a} + \dfrac{x}{b} = 1$.

Answer: $x = \dfrac{ab}{a+b}$

$bx + ax = ab$	*Multiply every term by* ab
$x(a+b) = ab$	*Factorise out* x
$x = \dfrac{ab}{a+b}$	

Marks M1 Multiplies by ab: $bx + ax = ab$ M1 $x(a+b) = ab$ A1 $x = \dfrac{ab}{a+b}$

21 Make x the subject of $\sqrt{x^2 + a} = b$, where x is positive.

Answer: $x = \sqrt{b^2 - a}$

$x^2 + a = b^2$	*Square both sides to remove the root*
$x^2 = b^2 - a$	
$x = \sqrt{b^2 - a}$	*Square root the whole of the right-hand side*

Marks M1 $x^2 + a = b^2$ M1 $x^2 = b^2 - a$ A1 $x = \sqrt{b^2 - a}$