Linear Inequalities: answers

Worked solutions and mark scheme · GCSE Maths · Higher tier

M = method mark, A = accuracy mark (needs the method), B = independent mark, ft = follow through from an earlier wrong answer.

Level 1 · Warm-up 14 marks

One- and two-step inequalities, listing integers in a range, and number lines.

1 Solve $3x - 5 > 7$.

Answer: $x > 4$

$3x > 12$	*Add 5 to both sides*
$x > 4$	*Divide both sides by 3: positive, so the sign stays the same*

Marks M1 $3x > 12$ A1 $x > 4$

2 Solve $\frac{x}{2} + 3 \leq 8$.

Answer: $x \leq 10$

$\frac{x}{2} \leq 5$	*Subtract 3 from both sides*
$x \leq 10$	*Multiply both sides by 2*

Marks M1 $\frac{x}{2} \leq 5$ A1 $x \leq 10$

3 Solve $4(x + 1) < 20$.

Answer: $x < 4$

$x + 1 < 5$	*Divide both sides by 4*
$x < 4$	

Marks M1 $4x + 4 < 20$ or $x + 1 < 5$ A1 $x < 4$

4 Write down all the integers n such that $-2 \leq n < 3$.

Answer: $-2,\ -1,\ 0,\ 1,\ 2$

$-2 \leq n \Rightarrow -2$ is included	$\leq$ *includes the end*
$n < 3 \Rightarrow 3$ is not included	$<$ *does not include the end*
$-2,\ -1,\ 0,\ 1,\ 2$	

Marks **B2** All five correct and no extras

5 Solve $5x + 2 \geq 2x + 17$.

Answer: $x \geq 5$

$3x + 2 \geq 17$	*Subtract $2x$ from both sides*
$3x \geq 15 \Rightarrow x \geq 5$	

Marks **M1** $3x \geq 15$ **A1** $x \geq 5$

6 Write down the inequality shown on the number line.



Answer: $-1 < x \leq 4$

$-1 < x$	*The open circle at -1: -1 itself is not included*
$x \leq 4$	*The filled circle at 4: 4 is included*
$-1 < x \leq 4$	

Marks **B2** $-1 < x \leq 4$ (B1 for $-1 < x$ or $x \leq 4$, or for both numbers with one sign wrong)

7 Show the inequality $-2 \leq x < 3$ on the number line.

Answer: A filled circle at -2 and an open circle at 3 with a line joining them



$-2 \leq x$	*-2 is included: a filled circle*
$x < 3$	*3 is not included: an open circle*

Marks B1 A filled circle at -2 and an open circle at 3 B1 A line joining them

Level 2 · Build

18 marks

Negative coefficients, double inequalities, number lines and a first look at set notation.

8 Solve $7 - 2x > 1$.

Answer: $x < 3$

$-2x > -6$	*Subtract 7 from both sides*
$x < 3$	*Divide by -2: a negative, so flip the sign*

Marks M1 $-2x > -6$ or $6 > 2x$ A1 $x < 3$

9 Solve $-1 < 2x + 3 \leq 9$.

Answer: $-2 < x \leq 3$

$-4 < 2x \leq 6$	*Subtract 3 from all three parts*
$-2 < x \leq 3$	*Divide all three parts by 2*

Marks M1 $-4 < 2x \leq 6$ A1 $-2 < x$ A1 $x \leq 3$, with both parts combined: $-2 < x \leq 3$

10 List all the integers x that satisfy $5 \leq 3x - 1 < 14$.

Answer: $2,\ 3,\ 4$

$6 \leq 3x < 15$	*Add 1 to all three parts*
$2 \leq x < 5$	*Divide by 3*
$2,\ 3,\ 4$	*2 is included; 5 is not*

Marks M1 $6 \leq 3x < 15$ A1 $2 \leq x < 5$ A1 $2,\ 3,\ 4$

11 Find the smallest integer that satisfies $4x - 3 > 11$.

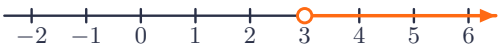
Answer: 4

$4x > 14 \Rightarrow x > 3.5$	
smallest integer $= 4$	*3 is too small, because $3 < 3.5$*

Marks M1 $x > 3.5$ A1 4

12 Solve $3x + 2 > 11$, and show your solution on the number line.

Answer: $x > 3$: an open circle at 3, with an arrow to the right



$3x > 9$	*Subtract 2 from both sides*
$x > 3$	*Divide both sides by 3*
open circle at 3	*3 itself is not included, but every number above it is*

Marks M1 $3x > 9$ A1 $x > 3$ B1 An open circle at 3 and an arrow to the right (ft their inequality)

13 Solve $\frac{x+1}{3} - \frac{x-2}{4} \geq 1$.

Answer: $x \geq 2$

$4(x+1) - 3(x-2) \geq 12$	*Multiply every term by 12 (positive, so the sign stays)*
$4x + 4 - 3x + 6 \geq 12$	*Careful: $-3(x-2) = -3x + 6$*
$x + 10 \geq 12 \Rightarrow x \geq 2$	

Marks M1 Multiplies every term by 12: $4(x+1) - 3(x-2) \geq 12$ M1 $x + 10 \geq 12$ A1 $x \geq 2$

14 Solve $3x + 1 > 7$. Give your answer using set notation.

Answer: $\{x : x > 2\}$

$3x > 6 \Rightarrow x > 2$

$\{x : x > 2\}$ *Read as "the set of all x such that $x > 2$"*

Marks M1 $x > 2$ A1 $\{x : x > 2\}$

Level 3 · Exam-style — 23 marks

Integer solutions to two inequalities, contexts and spotting mistakes.

15 n is an integer. $2n + 5 > 9$ and $3n - 4 < 14$. Find all the possible values of n.

Answer: $3,\ 4,\ 5$

$2n > 4 \Rightarrow n > 2$

$3n < 18 \Rightarrow n < 6$

$2 < n < 6 \Rightarrow n = 3,\ 4,\ 5$ *Neither end is included*

Marks M1 $n > 2$ M1 $n < 6$ A1 $3,\ 4,\ 5$

16 A taxi charges £4 plus £2 for every mile. Ella has £25. What is the greatest whole number of miles she can travel?

Answer: 10 miles

$4 + 2m \leq 25$ *The cost must be at most £25*

$2m \leq 21 \Rightarrow m \leq 10.5$

greatest whole number $= 10$ *11 miles would cost £26*

Marks M1 $4 + 2m \leq 25$ A1 $m \leq 10.5$ A1 10

17 Tom solves $6 - 3x < 12$. He writes $-3x < 6$, so $x < -2$. Explain Tom's mistake, and give the correct answer.

Answer: Dividing by -3 means the sign must be turned round. $x > -2$.

$-3x < 6$ *Tom is right so far*

$x > -2$ *Dividing by -3 flips $<$ to $>$*

Marks B1 Explains that dividing by a negative number reverses the inequality B1 $x > -2$

18 A rectangle has sides of length x cm and $(x+4)$ cm. Its perimeter is less than 40 cm. Find the range of possible values of x.

Answer: $0 < x < 8$

$2x + 2(x+4) < 40$ — *Perimeter: two of each side*

$4x + 8 < 40 \Rightarrow 4x < 32 \Rightarrow x < 8$

$0 < x < 8$ — *A length must also be more than 0*

Marks M1 $2x + 2(x+4) < 40$ oe A1 $x < 8$ B1 $x > 0$, because a length must be positive

19 (a) The number line shows the solution of an inequality. Write down all the integers it shows.



Answer: $-1, 0, 1, 2, 3$

$-2 < x \leq 3$ — *Open at -2, filled at 3*

$-1,\ 0,\ 1,\ 2,\ 3$ — *-2 is not included, but 3 is*

Marks B2 $-1, 0, 1, 2, 3$ (B1 for four correct and none wrong, or all five and one extra)

(b) Show that the solution of $4 < 2x + 8 \leq 14$ is the inequality shown on the number line.

Answer: $-4 < 2x \leq 6$, so $-2 < x \leq 3$: the inequality shown.

$4 - 8 < 2x \leq 14 - 8$ — *Subtract 8 from all three parts*

$-4 < 2x \leq 6$

$-2 < x \leq 3$ — *Divide all three parts by 2: the same as the number line*

Marks M1 $-4 < 2x \leq 6$ A1 $-2 < x \leq 3$ A1 Matches the number line: open circle at -2, filled circle at 3

20 Find the integers that satisfy both $3x - 2 > x + 3$ and $5x \leq 2x + 21$.

Answer: $3,\ 4,\ 5,\ 6,\ 7$

$3x - 2 > x + 3 \Rightarrow 2x > 5 \Rightarrow x > 2.5$

$5x \leq 2x + 21 \Rightarrow 3x \leq 21 \Rightarrow x \leq 7$

$2.5 < x \leq 7 \Rightarrow 3,\ 4,\ 5,\ 6,\ 7$ — *7 is included; 2.5 is not a whole number anyway*

Marks M1 $2x > 5$ A1 $x > 2.5$ A1 $x \leq 7$ A1 $3,\ 4,\ 5,\ 6,\ 7$

21 Solve $-3 \leq 2x - 5 < 4$. Give your answer using set notation.

Answer: $\{x : 1 \leq x < 4.5\}$

$2 \leq 2x < 9$	*Add 5 to all three parts*
$1 \leq x < 4.5$	*Divide all three parts by 2*
$\{x : 1 \leq x < 4.5\}$	

Marks **M1** $2 \leq 2x < 9$ **A1** $1 \leq x < 4.5$ **A1** $\{x : 1 \leq x < 4.5\}$

Level 4 · Challenge — 13 marks

Grade 8–9: two variables, chains of inequalities, unknown constants and set notation.

22 x and y are integers. $y \geq 1$, $x + y \leq 6$ and $y < x$. List all the possible pairs $(x,\ y)$.

Answer: $(2, 1)$, $(3, 1)$, $(4, 1)$, $(5, 1)$, $(3, 2)$, $(4, 2)$

$y = 1:\quad x > 1$ and $x \leq 5 \Rightarrow x = 2, 3, 4, 5$	*Work through each possible value of y*
$y = 2:\quad x > 2$ and $x \leq 4 \Rightarrow x = 3, 4$	
$y = 3:\quad x > 3$ and $x \leq 3 \Rightarrow$ none	*Larger y values give no pairs either*

Marks **B3** All six pairs and no extras (B2 for at least four correct pairs with at most one wrong; B1 for at least two correct pairs)

23 Solve $3 - 2x \leq x + 12 < 4x + 3$. Give your answer using set notation.

Answer: $\{x : x > 3\}$

$3 - 2x \leq x + 12 \Rightarrow -9 \leq 3x \Rightarrow x \geq -3$	*The x is in all three parts, so solve each half separately*
$x + 12 < 4x + 3 \Rightarrow 9 < 3x \Rightarrow x > 3$	
$\{x : x > 3\}$	*Every $x > 3$ also satisfies $x \geq -3$*

Marks **M1** Splits into $3 - 2x \leq x + 12$ and $x + 12 < 4x + 3$ **A1** $x \geq -3$ **A1** $x > 3$ **A1** $\{x : x > 3\}$, the values that satisfy both

24 A rectangle has sides $(x + 2)$ cm and 5 cm. The number of square centimetres in its area is greater than the number of centimetres in its perimeter. Find the range of possible values of x.

Answer: $x > \frac{4}{3}$

$5(x + 2) > 2(x + 2) + 2(5)$	*Area > perimeter*
$5x + 10 > 2x + 14$	
$3x > 4 \Rightarrow x > \frac{4}{3}$	*This also makes the side $x + 2$ positive*

Marks M1 $5(x + 2) > 2(x + 2) + 10$ M1 $5x + 10 > 2x + 14$ A1 $x > \frac{4}{3}$ oe

25 The solution of the inequality $3x - k > 2x + 5$ is $x > 9$. Find the value of the constant k.

Answer: $k = 4$

$3x - 2x > 5 + k \Rightarrow x > k + 5$	*Solve with k still in it*
$k + 5 = 9$	*Compare with $x > 9$*
$k = 4$	

Marks M1 $x > k + 5$ M1 $k + 5 = 9$ A1 $k = 4$