Linear Inequalities: answers

Worked solutions and mark scheme · GCSE Maths · Foundation tier

M = method mark, A = accuracy mark (needs the method), B = independent mark, ft = follow through from an earlier wrong answer.

Level 1 · Warm-up 10 marks

One- and two-step inequalities, and listing integers in a range.

1 Solve $3x - 5 > 7$.

Answer: $x > 4$

$3x > 12$	*Add 5 to both sides*
$x > 4$	*Divide both sides by 3: positive, so the sign stays the same*

Marks M1 $3x > 12$ A1 $x > 4$

2 Solve $\frac{x}{2} + 3 \leq 8$.

Answer: $x \leq 10$

$\frac{x}{2} \leq 5$	*Subtract 3 from both sides*
$x \leq 10$	*Multiply both sides by 2*

Marks M1 $\frac{x}{2} \leq 5$ A1 $x \leq 10$

3 Solve $4(x + 1) < 20$.

Answer: $x < 4$

$x + 1 < 5$	*Divide both sides by 4*
$x < 4$	

Marks M1 $4x + 4 < 20$ or $x + 1 < 5$ A1 $x < 4$

4 Write down all the integers n such that $-2 \leq n < 3$.

Answer: $-2,\ -1,\ 0,\ 1,\ 2$

$-2 \leq n \Rightarrow -2$ is included	*$\leq$ includes the end*
$n < 3 \Rightarrow 3$ is not included	*$<$ does not include the end*
$-2,\ -1,\ 0,\ 1,\ 2$	

Marks **B2** All five correct and no extras

5 Solve $5x + 2 \geq 2x + 17$.

Answer: $x \geq 5$

$3x + 2 \geq 17$	*Subtract $2x$ from both sides*
$3x \geq 15 \Rightarrow x \geq 5$	

Marks **M1** $3x \geq 15$ **A1** $x \geq 5$

Level 2 · Build

10 marks

Negative coefficients, double inequalities, and a first look at set notation.

6 Solve $7 - 2x > 1$.

Answer: $x < 3$

$-2x > -6$	*Subtract 7 from both sides*
$x < 3$	*Divide by -2: a negative, so flip the sign*

Marks **M1** $-2x > -6$ or $6 > 2x$ **A1** $x < 3$

7 Solve $-1 < 2x + 3 \leq 9$.

Answer: $-2 < x \leq 3$

$-4 < 2x \leq 6$	*Subtract 3 from all three parts*
$-2 < x \leq 3$	*Divide all three parts by 2*

Marks **M1** $-4 < 2x \leq 6$ **A1** $-2 < x$ **A1** $x \leq 3$, with both parts combined: $-2 < x \leq 3$

8 List all the integers x that satisfy $5 \leq 3x - 1 < 14$.

Answer: $2,\ 3,\ 4$

$6 \leq 3x < 15$	*Add* 1 *to all three parts*
$2 \leq x < 5$	*Divide by* 3
$2,\ 3,\ 4$	2 *is included;* 5 *is not*

Marks M1 $6 \leq 3x < 15$ A1 $2 \leq x < 5$ A1 $2,\ 3,\ 4$

9 Find the smallest integer that satisfies $4x - 3 > 11$.

Answer: 4

$4x > 14 \ \Rightarrow \ x > 3.5$	
smallest integer $= 4$	3 *is too small, because* $3 < 3.5$

Marks M1 $x > 3.5$ A1 4

Level 3 · Exam-style — 11 marks

Integer solutions to two inequalities, contexts and spotting mistakes.

10 n is an integer. $2n + 5 > 9$ and $3n - 4 < 14$. Find all the possible values of n.

Answer: $3,\ 4,\ 5$

$2n > 4 \ \Rightarrow \ n > 2$	
$3n < 18 \ \Rightarrow \ n < 6$	
$2 < n < 6 \ \Rightarrow \ n = 3,\ 4,\ 5$	*Neither end is included*

Marks M1 $n > 2$ M1 $n < 6$ A1 $3,\ 4,\ 5$

11 A taxi charges £4 plus £2 for every mile. Ella has £25. What is the greatest whole number of miles she can travel?

Answer: 10 miles

$4 + 2m \leq 25$	*The cost must be at most £25*
$2m \leq 21 \Rightarrow m \leq 10.5$	
greatest whole number $= 10$	*11 miles would cost £26*

Marks M1 $4 + 2m \leq 25$ A1 $m \leq 10.5$ A1 10

12 Tom solves $6 - 3x < 12$. He writes $-3x < 6$, so $x < -2$. Explain Tom's mistake, and give the correct answer.

Answer: Dividing by -3 means the sign must be turned round. $x > -2$.

$-3x < 6$	*Tom is right so far*
$x > -2$	*Dividing by -3 flips $<$ to $>$*

Marks B1 Explains that dividing by a negative number reverses the inequality B1 $x > -2$

13 A rectangle has sides of length x cm and $(x + 4)$ cm. Its perimeter is less than 40 cm. Find the range of possible values of x.

Answer: $0 < x < 8$

$2x + 2(x + 4) < 40$	*Perimeter: two of each side*
$4x + 8 < 40 \Rightarrow 4x < 32 \Rightarrow x < 8$	
$0 < x < 8$	*A length must also be more than 0*

Marks M1 $2x + 2(x + 4) < 40$ oe A1 $x < 8$ B1 $x > 0$, because a length must be positive

Level 4 · Challenge (Foundation) — 9 marks

Grade 4–5 problem solving: inequalities from budgets, shapes and number puzzles.

14 A school trip costs £300 for the coach plus £12 for each student. The budget is at most £800. What is the greatest number of students who can go?

Answer: 41 students

$300 + 12n \leq 800$

$12n \leq 500 \Rightarrow n \leq 41.66\ldots$

greatest number $= 41$

Round down: 42 students would cost £804

Marks M1 $300 + 12n \leq 800$ A1 $n \leq 41.6\ldots$ A1 41

15 Sam thinks of a whole number. Three times the number, subtract 4, is more than 11. Twice the number, add 1, is less than 15. What is Sam's number?

Answer: 6

$3n - 4 > 11 \Rightarrow 3n > 15 \Rightarrow n > 5$

$2n + 1 < 15 \Rightarrow 2n < 14 \Rightarrow n < 7$

$5 < n < 7 \Rightarrow n = 6$

Marks M1 $3n - 4 > 11$ and $2n + 1 < 15$ A1 $n > 5$ and $n < 7$ A1 6

16 A rectangle has width x cm and length $2x$ cm. Its perimeter must be at least 24 cm and at most 42 cm. Find the range of possible values of x.

Answer: $4 \leq x \leq 7$

perimeter $= x + 2x + x + 2x = 6x$

$24 \leq 6x \leq 42$

"At least" means $\geq$; "at most" means $\leq$

$4 \leq x \leq 7$

Divide all three parts by 6

Marks M1 Perimeter $= 6x$ M1 $24 \leq 6x \leq 42$ A1 $4 \leq x \leq 7$