Factorising Quadratics: answers

Worked solutions and mark scheme · GCSE Maths · Higher tier

M = method mark, A = accuracy mark (needs the method), B = independent mark, ft = follow through from an earlier wrong answer.

Level 1 · Warm-up 9 marks

Quadratics $x^2 + bx + c$ with positive and negative numbers, and taking out x.

1 Factorise $x^2 + 7x + 10$.

Answer: $(x + 2)(x + 5)$

$2 \times 5 = 10, \quad 2 + 5 = 7$

$x^2 + 7x + 10 = (x + 2)(x + 5)$

Find two numbers that multiply to 10 and add to 7

Marks M1 $(x \pm 2)(x \pm 5)$ A1 $(x + 2)(x + 5)$

2 Factorise $x^2 + 5x$.

Answer: $x(x + 5)$

$x^2 + 5x = x(x + 5)$

Both terms have a common factor of x

Marks B1 $x(x + 5)$

3 Factorise $x^2 - 9x + 20$.

Answer: $(x - 4)(x - 5)$

$(-4) \times (-5) = 20, \quad (-4) + (-5) = -9$

$x^2 - 9x + 20 = (x - 4)(x - 5)$

Both numbers are negative: they multiply to $+20$ and add to -9

Marks M1 $(x \pm 4)(x \pm 5)$ A1 $(x - 4)(x - 5)$

4 Factorise $x^2 + 2x - 15$.

Answer: $(x + 5)(x - 3)$

$5 \times (-3) = -15, \quad 5 + (-3) = 2$

$x^2 + 2x - 15 = (x + 5)(x - 3)$

A negative product means one number is negative

Marks M1 $(x \pm 5)(x \pm 3)$ A1 $(x + 5)(x - 3)$

5 Factorise $x^2 + 10x + 24$.

Answer: $(x + 4)(x + 6)$

$4 \times 6 = 24, \quad 4 + 6 = 10$

$x^2 + 10x + 24 = (x + 4)(x + 6)$

List factor pairs of 24: $1, 24$; $2, 12$; $3, 8$; $4, 6$

Marks M1 $(x \pm 4)(x \pm 6)$ A1 $(x + 4)(x + 6)$

Level 2 · Build

11 marks

Harder signs, the difference of two squares, perfect squares and a first look at $ax^2 + bx + c$.

6 Factorise $x^2 - 3x - 28$.

Answer: $(x - 7)(x + 4)$

$(-7) \times 4 = -28, \quad (-7) + 4 = -3$

$x^2 - 3x - 28 = (x - 7)(x + 4)$

Opposite signs; the bigger number is negative because b is negative

Marks M1 $(x \pm 7)(x \pm 4)$ A1 $(x - 7)(x + 4)$

7 Factorise $x^2 - 11x + 24$.

Answer: $(x - 3)(x - 8)$

$(-3) \times (-8) = 24, \quad (-3) + (-8) = -11$

$x^2 - 11x + 24 = (x - 3)(x - 8)$

Positive c and negative b: both numbers are negative

Marks M1 $(x \pm 3)(x \pm 8)$ A1 $(x - 3)(x - 8)$

8 Factorise $x^2 - 64$.

Answer: $(x + 8)(x - 8)$

$x^2 - 64 = x^2 - 8^2$

$= (x + 8)(x - 8)$

Difference of two squares: $a^2 - b^2 = (a + b)(a - b)$

Marks B1 $(x + 8)(x - 8)$ — accept in either order

9 Factorise $x^2 + 14x + 49$.

Answer: $(x + 7)^2$

$7 \times 7 = 49, \quad 7 + 7 = 14$

$x^2 + 14x + 49 = (x + 7)(x + 7) = (x + 7)^2$

The same number twice, so it is a perfect square

Marks M1 $(x \pm 7)(x \pm 7)$ A1 $(x + 7)^2$ — allow $(x + 7)(x + 7)$

10 Factorise $2x^2 + 7x + 3$.

Answer: $(2x + 1)(x + 3)$

$ac = 2 \times 3 = 6; \quad 1 \times 6 = 6,\ 1 + 6 = 7$

Find two numbers that multiply to ac and add to b

$2x^2 + x + 6x + 3$

Split the middle term using 1 and 6

$= x(2x + 1) + 3(2x + 1)$

Factorise each pair: the brackets match

$= (2x + 1)(x + 3)$

Marks M1 $(2x \pm 1)(x \pm 3)$, or a correct split of the middle term, e.g. $2x^2 + x + 6x + 3$ A1 $(2x + 1)(x + 3)$

11 Factorise $3x^2 - 10x + 8$.

Answer: $(3x - 4)(x - 2)$

$ac = 24; \quad (-4) \times (-6) = 24,\ (-4) + (-6) = -10$	*Both negative, because b is negative and ac is positive*
$3x^2 - 4x - 6x + 8$	
$= x(3x - 4) - 2(3x - 4)$	*Take out -2 so the second bracket matches*
$= (3x - 4)(x - 2)$	

Marks **M1** $(3x \pm 4)(x \pm 2)$, or a correct split of the middle term **A1** $(3x - 4)(x - 2)$

Level 3 · Exam-style — 15 marks

Factorising fully, "hence" questions and spotting mistakes, in real GCSE style.

12 Factorise fully $3x^2 + 12x - 15$.

Answer: $3(x + 5)(x - 1)$

$3x^2 + 12x - 15 = 3(x^2 + 4x - 5)$	*Take out the common factor 3 first*
$5 \times (-1) = -5, \quad 5 + (-1) = 4$	
$= 3(x + 5)(x - 1)$	

Marks **M1** $3(x^2 + 4x - 5)$, or $(3x \pm 15)(x \pm 1)$, or $(3x \pm 3)(x \pm 5)$ **A1** $3(x + 5)(x - 1)$

13 (a) Factorise $x^2 - 9$.

Answer: $(x + 3)(x - 3)$

$x^2 - 9 = x^2 - 3^2 = (x + 3)(x - 3)$	*Difference of two squares*

Marks **B1** $(x + 3)(x - 3)$

(b) Hence work out the value of $103^2 - 9$. You must not use a calculator, and you must show your working.

Answer: $10\,600$

$103^2 - 9 = (103 + 3)(103 - 3)$ *Use part (a) with $x = 103$*

$= 106 \times 100$

$= 10\,600$

Marks M1 $(103 + 3)(103 - 3)$ or 106×100 A1 $10\,600$

14 Sam factorises $x^2 - 5x - 14$ and writes $(x - 7)(x - 2)$. Explain Sam's mistake, and factorise $x^2 - 5x - 14$ correctly.

Answer: $(x - 7)(x + 2)$. Sam's numbers multiply to $+14$, not -14.

$(-7) \times (-2) = +14 \neq -14$ *Sam's brackets give the wrong sign for c*

$(-7) \times 2 = -14, \quad (-7) + 2 = -5$ *A negative c needs opposite signs*

$x^2 - 5x - 14 = (x - 7)(x + 2)$

Marks B1 Explains that $(-7) \times (-2) = +14$, or that the numbers need opposite signs B1 $(x - 7)(x + 2)$

15 (a) A rectangle has area $(x^2 + 11x + 28)$ cm^2. Its width is $(x + 4)$ cm. Find an expression for its length.

Answer: $x + 7$ cm

$x^2 + 11x + 28 = (x + 4)(x + 7)$ *Factorise the area: one bracket must be the width*

length $= x + 7$ *Area = width × length*

Marks M1 $(x + 4)(x + 7)$ A1 $x + 7$

(b) Find an expression for the perimeter of the rectangle. Give your answer in its simplest form.

Answer: $4x + 22$ cm

$2(x + 4) + 2(x + 7)$ *Two widths and two lengths*

$= 2x + 8 + 2x + 14$

$= 4x + 22$

Marks M1 $2(x + 4) + 2(x + 7)$ oe — ft their length A1 $4x + 22$

16 Factorise $6x^2 + x - 2$.

Answer: $(2x - 1)(3x + 2)$

$ac = -12; \quad 4 \times (-3) = -12,\ 4 + (-3) = 1$

Two numbers that multiply to -12 and add to 1

$6x^2 + 4x - 3x - 2$
$= 2x(3x + 2) - 1(3x + 2)$
$= (2x - 1)(3x + 2)$

Marks **M1** $(2x \pm 1)(3x \pm 2)$, or a correct split of the middle term, e.g. $6x^2 + 4x - 3x - 2$ **A1** $(2x - 1)(3x + 2)$

17 Factorise fully $12x^2 - 27$.

Answer: $3(2x + 3)(2x - 3)$

$12x^2 - 27 = 3(4x^2 - 9)$
$= 3\left((2x)^2 - 3^2\right)$
$= 3(2x + 3)(2x - 3)$

Common factor 3 first

$4x^2 - 9$ is a difference of two squares

Marks **M1** $3(4x^2 - 9)$, or $(6x \pm 9)(2x \pm 3)$, or $(2x \pm 3)(6x \pm 9)$ **A1** $3(2x + 3)(2x - 3)$

Level 4 · Challenge 12 marks
Grade 8–9: algebraic difference of two squares, proof and unknown coefficients.

18 Factorise fully $(x + 3)^2 - (x - 1)^2$.

Answer: $8(x + 1)$

$= \left[(x + 3) + (x - 1)\right]\left[(x + 3) - (x - 1)\right]$

$= (2x + 2)(4)$

$= 8x + 8 = 8(x + 1)$

Difference of two squares with $a = x + 3$ and $b = x - 1$

Careful: $-(x - 1) = -x + 1$

"Fully" means take out the common factor 8

Marks **M1** $\left[(x + 3) + (x - 1)\right]\left[(x + 3) - (x - 1)\right]$, or both brackets expanded correctly **M1** $(2x + 2) \times 4$ or $8x + 8$ **A1** $8(x + 1)$

19 Prove that the difference between the squares of any two consecutive odd numbers is a multiple of 8.

Answer: $(2n+1)^2-(2n-1)^2=8n$, which is a multiple of 8.

Let the numbers be $2n-1$ and $2n+1$	*n is any whole number; odd numbers are 2 apart*
$(2n+1)^2-(2n-1)^2=$ $\big[(2n+1)+(2n-1)\big]\big[(2n+1)-(2n-1)\big]$ $=4n\times 2=8n$	*Difference of two squares*
$8n=8\times n$, a multiple of 8	*State the conclusion: it earns the final mark*

Marks **B1** Consecutive odd numbers written as $2n-1$ and $2n+1$ (or $2n+1$ and $2n+3$) **M1** $\big[(2n+1)+(2n-1)\big]\big[(2n+1)-(2n-1)\big]$, or both squares expanded correctly **A1** $8n$ with a conclusion that it is a multiple of 8

20 $3x^2+bx-10\equiv(3x+5)(x+c)$. Find the value of b and the value of c.

Answer: $b=-1,\ c=-2$

$(3x+5)(x+c)=3x^2+(3c+5)x+5c$	*Expand the right-hand side*
$5c=-10\ \Rightarrow\ c=-2$	*Compare the constant terms*
$b=3c+5=-6+5=-1$	*Compare the x terms*

Marks **M1** Expands $(3x+5)(x+c)$ to $3x^2+3cx+5x+5c$ **M1** $5c=-10$ **A1** $c=-2$ and $b=-1$

21 (a) Factorise $4x^2-25$.

Answer: $(2x+5)(2x-5)$

$4x^2-25=(2x)^2-5^2=(2x+5)(2x-5)$	*Difference of two squares with $a=2x$*

Marks **B1** $(2x+5)(2x-5)$

(b) Hence, or otherwise, factorise fully $4x^4-25x^2$.

Answer: $x^2(2x+5)(2x-5)$

$4x^4-25x^2=x^2(4x^2-25)$	*Take out the common factor x^2 first*
$=x^2(2x+5)(2x-5)$	*Use part (a)*

Marks **M1** $x^2(4x^2-25)$ **A1** $x^2(2x+5)(2x-5)$