Factorising Quadratics: answers

Worked solutions and mark scheme · GCSE Maths · Foundation tier

M = method mark, A = accuracy mark (needs the method), B = independent mark, ft = follow through from an earlier wrong answer.

Level 1 · Warm-up 9 marks

Quadratics $x^2 + bx + c$ with positive and negative numbers, and taking out x.

1 Factorise $x^2 + 7x + 10$.

Answer: $(x + 2)(x + 5)$

$2 \times 5 = 10, \quad 2 + 5 = 7$

$x^2 + 7x + 10 = (x + 2)(x + 5)$

Find two numbers that multiply to 10 and add to 7

Marks M1 $(x \pm 2)(x \pm 5)$ A1 $(x + 2)(x + 5)$

2 Factorise $x^2 + 5x$.

Answer: $x(x + 5)$

$x^2 + 5x = x(x + 5)$

Both terms have a common factor of x

Marks B1 $x(x + 5)$

3 Factorise $x^2 - 9x + 20$.

Answer: $(x - 4)(x - 5)$

$(-4) \times (-5) = 20, \quad (-4) + (-5) = -9$

$x^2 - 9x + 20 = (x - 4)(x - 5)$

Both numbers are negative: they multiply to $+20$ and add to -9

Marks M1 $(x \pm 4)(x \pm 5)$ A1 $(x - 4)(x - 5)$

4 Factorise $x^2 + 2x - 15$.

Answer: $(x + 5)(x - 3)$

$5 \times (-3) = -15, \quad 5 + (-3) = 2$

$x^2 + 2x - 15 = (x + 5)(x - 3)$

A negative product means one number is negative

Marks M1 $(x \pm 5)(x \pm 3)$ A1 $(x + 5)(x - 3)$

5 Factorise $x^2 + 10x + 24$.

Answer: $(x + 4)(x + 6)$

$4 \times 6 = 24, \quad 4 + 6 = 10$

$x^2 + 10x + 24 = (x + 4)(x + 6)$

List factor pairs of 24: $1, 24$; $2, 12$; $3, 8$; $4, 6$

Marks M1 $(x \pm 4)(x \pm 6)$ A1 $(x + 4)(x + 6)$

Level 2 · Build

7 marks

Harder signs, the difference of two squares, perfect squares and a first look at $ax^2 + bx + c$.

6 Factorise $x^2 - 3x - 28$.

Answer: $(x - 7)(x + 4)$

$(-7) \times 4 = -28, \quad (-7) + 4 = -3$

$x^2 - 3x - 28 = (x - 7)(x + 4)$

Opposite signs; the bigger number is negative because b is negative

Marks M1 $(x \pm 7)(x \pm 4)$ A1 $(x - 7)(x + 4)$

7 Factorise $x^2 - 11x + 24$.

Answer: $(x - 3)(x - 8)$

$(-3) \times (-8) = 24, \quad (-3) + (-8) = -11$

$x^2 - 11x + 24 = (x - 3)(x - 8)$

Positive c and negative b: both numbers are negative

Marks M1 $(x \pm 3)(x \pm 8)$ A1 $(x - 3)(x - 8)$

8 Factorise $x^2 - 64$.

Answer: $(x + 8)(x - 8)$

$$x^2 - 64 = x^2 - 8^2$$
$$= (x + 8)(x - 8)$$

Difference of two squares: $a^2 - b^2 = (a + b)(a - b)$

Marks B1 $(x + 8)(x - 8)$ — accept in either order

9 Factorise $x^2 + 14x + 49$.

Answer: $(x + 7)^2$

$$7 \times 7 = 49, \quad 7 + 7 = 14$$
$$x^2 + 14x + 49 = (x + 7)(x + 7) = (x + 7)^2$$

The same number twice, so it is a perfect square

Marks M1 $(x \pm 7)(x \pm 7)$ A1 $(x + 7)^2$ — allow $(x + 7)(x + 7)$

Level 3 · Exam-style — 11 marks

Factorising fully, "hence" questions and spotting mistakes, in real GCSE style.

10 Factorise fully $3x^2 + 12x - 15$.

Answer: $3(x + 5)(x - 1)$

$$3x^2 + 12x - 15 = 3(x^2 + 4x - 5)$$
$$5 \times (-1) = -5, \quad 5 + (-1) = 4$$
$$= 3(x + 5)(x - 1)$$

Take out the common factor 3 first

Marks M1 $3(x^2 + 4x - 5)$, or $(3x \pm 15)(x \pm 1)$, or $(3x \pm 3)(x \pm 5)$ A1 $3(x + 5)(x - 1)$

11 (a) Factorise $x^2 - 9$.

Answer: $(x + 3)(x - 3)$

$$x^2 - 9 = x^2 - 3^2 = (x + 3)(x - 3)$$

Difference of two squares

Marks B1 $(x + 3)(x - 3)$

(b) Hence work out the value of $103^2 - 9$. You must not use a calculator, and you must show your working.

Answer: $10\,600$

$103^2 - 9 = (103 + 3)(103 - 3)$ — *Use part (a) with $x = 103$*

$= 106 \times 100$

$= 10\,600$

Marks M1 $(103 + 3)(103 - 3)$ or 106×100 A1 $10\,600$

12 Sam factorises $x^2 - 5x - 14$ and writes $(x - 7)(x - 2)$. Explain Sam's mistake, and factorise $x^2 - 5x - 14$ correctly.

Answer: $(x - 7)(x + 2)$. Sam's numbers multiply to $+14$, not -14.

$(-7) \times (-2) = +14 \neq -14$ — *Sam's brackets give the wrong sign for c*

$(-7) \times 2 = -14, \quad (-7) + 2 = -5$ — *A negative c needs opposite signs*

$x^2 - 5x - 14 = (x - 7)(x + 2)$

Marks B1 Explains that $(-7) \times (-2) = +14$, or that the numbers need opposite signs B1 $(x - 7)(x + 2)$

13 (a) A rectangle has area $(x^2 + 11x + 28)$ cm^2. Its width is $(x + 4)$ cm. Find an expression for its length.

Answer: $x + 7$ cm

$x^2 + 11x + 28 = (x + 4)(x + 7)$ — *Factorise the area: one bracket must be the width*

length $= x + 7$ — *Area = width × length*

Marks M1 $(x + 4)(x + 7)$ A1 $x + 7$

(b) Find an expression for the perimeter of the rectangle. Give your answer in its simplest form.

Answer: $4x + 22$ cm

$2(x + 4) + 2(x + 7)$ — *Two widths and two lengths*

$= 2x + 8 + 2x + 14$

$= 4x + 22$

Marks M1 $2(x + 4) + 2(x + 7)$ oe — ft their length A1 $4x + 22$

Level 4 · Challenge (Foundation) 9 marks
Grade 4–5 problem solving: using factorising to calculate and to explain.

14 Use factorising to work out $87^2 - 13^2$. You must not use a calculator, and you must show your working.

Answer: 7400

$$87^2 - 13^2 = (87 + 13)(87 - 13)$$
$$= 100 \times 74$$
$$= 7400$$

Difference of two squares: $a^2 - b^2 = (a + b)(a - b)$

Marks M1 $(87 + 13)(87 - 13)$ or 100×74 A1 7400

15 (a) Factorise $x^2 + 6x + 9$.

Answer: $(x + 3)^2$

$$3 \times 3 = 9, \quad 3 + 3 = 6$$
$$x^2 + 6x + 9 = (x + 3)^2$$

Marks M1 $(x \pm 3)(x \pm 3)$ A1 $(x + 3)^2$ — allow $(x + 3)(x + 3)$

(b) Hence explain why $x^2 + 6x + 9$ can never be negative.

Answer: $x^2 + 6x + 9 = (x + 3)^2$, and a square is never negative, so $x^2 + 6x + 9 \geq 0$.

$$x^2 + 6x + 9 = (x + 3)^2$$
$$(x + 3)^2 \geq 0$$

From part (a)

Squaring any number, positive or negative, never gives a negative answer

Marks B1 States that $(x + 3)^2$ is a square, so it is always ≥ 0 — ft their (a) if it is a square

16 (a) $x^2 - 10x + 21$ can be written as $(x - a)(x - b)$, where a and b are whole numbers and $a < b$. Find the values of a and b.

Answer: $a = 3,\ b = 7$

$(-3) \times (-7) = 21, \quad (-3) + (-7) = -10$

$x^2 - 10x + 21 = (x - 3)(x - 7)$

$a = 3, \quad b = 7$ *The question says $a < b$*

Marks M1 $(x - 3)(x - 7)$ A1 $a = 3$ and $b = 7$

(b) Hence work out the value of $x^2 - 10x + 21$ when $x = 107$. You must not use a calculator.

Answer: $10\,400$

$(107 - 3)(107 - 7)$ *Substitute into the factorised form: much easier than 107^2*

$= 104 \times 100$

$= 10\,400$

Marks M1 $(107 - 3)(107 - 7)$ or 104×100 — ft their a and b A1 $10\,400$