Algebraic Fractions: answers

Worked solutions and mark scheme · GCSE Maths · Higher tier

M = method mark, A = accuracy mark (needs the method), B = independent mark, ft = follow through from an earlier wrong answer.

Level 1 · Warm-up — 10 marks

Cancelling common factors, and adding simple fractions.

1 Simplify $\frac{6x^2}{9x}$.

Answer: $\frac{2x}{3}$

$$\frac{6x^2}{9x} = \frac{6 \times x \times x}{9 \times x}$$
$$= \frac{2x}{3}$$

Write out the factors

Cancel 3 and x from the top and bottom

Marks B1 $\frac{2x}{3}$ oe

2 Simplify $\frac{3x+6}{x+2}$.

Answer: 3

$$\frac{3x+6}{x+2} = \frac{3(x+2)}{x+2}$$
$$= 3$$

Factorise the top

Cancel the common factor $(x+2)$

Marks M1 $3(x+2)$ A1 3

3 Simplify $\frac{x^2+5x}{x^2-25}$.

Answer: $\frac{x}{x-5}$

$$\frac{x^2+5x}{x^2-25} = \frac{x(x+5)}{(x+5)(x-5)}$$
$$= \frac{x}{x-5}$$

Common factor on top; difference of two squares on the bottom

Cancel $(x+5)$

Marks M1 $x(x+5)$ M1 $(x+5)(x-5)$ A1 $\frac{x}{x-5}$

4 Simplify $\frac{x^2+3x+2}{x+1}$.

Answer: $x+2$

$$\frac{x^2+3x+2}{x+1} = \frac{(x+1)(x+2)}{x+1}$$
Factorise the quadratic on top

$$= x+2$$

Marks M1 $(x+1)(x+2)$ A1 $x+2$

5 Write $\frac{x}{3}+\frac{x}{4}$ as a single fraction.

Answer: $\frac{7x}{12}$

$$\frac{x}{3}+\frac{x}{4} = \frac{4x}{12}+\frac{3x}{12}$$
Common denominator 12

$$= \frac{7x}{12}$$

Marks M1 $\frac{4x}{12}+\frac{3x}{12}$ A1 $\frac{7x}{12}$

Level 2 · Build

15 marks

Factorising quadratics to simplify, and adding, subtracting and multiplying.

6 Simplify fully $\frac{x^2-9}{x^2+x-6}$.

Answer: $\frac{x-3}{x-2}$

$$\frac{x^2-9}{x^2+x-6} = \frac{(x+3)(x-3)}{(x+3)(x-2)}$$
Factorise both the top and the bottom

$$= \frac{x-3}{x-2}$$
Cancel $(x+3)$

Marks M1 $(x+3)(x-3)$ M1 $(x+3)(x-2)$ A1 $\frac{x-3}{x-2}$

7 Write $\frac{2}{x} + \frac{3}{x+1}$ as a single fraction.

Answer: $\frac{5x+2}{x(x+1)}$

$$\frac{2}{x} + \frac{3}{x+1} = \frac{2(x+1)}{x(x+1)} + \frac{3x}{x(x+1)}$$
Multiply each top by the other bottom

$$= \frac{2x+2+3x}{x(x+1)}$$

$$= \frac{5x+2}{x(x+1)}$$

Marks M1 Common denominator $x(x+1)$ M1 $2(x+1)+3x$ A1 $\frac{5x+2}{x(x+1)}$ — allow $\frac{5x+2}{x^2+x}$

8 Simplify fully $\frac{2x^2+7x+3}{4x^2-1}$.

Answer: $\frac{x+3}{2x-1}$

$2x^2+7x+3 = (2x+1)(x+3)$
$ac = 6$: use 1 and 6

$4x^2-1 = (2x+1)(2x-1)$
Difference of two squares

$$\frac{(2x+1)(x+3)}{(2x+1)(2x-1)} = \frac{x+3}{2x-1}$$

Marks M1 $(2x+1)(x+3)$ M1 $(2x+1)(2x-1)$ A1 $\frac{x+3}{2x-1}$

9 Simplify fully $\frac{x+2}{x-1} \times \frac{x^2-1}{x^2+2x}$.

Answer: $\frac{x+1}{x}$

$$\frac{x+2}{x-1} \times \frac{(x+1)(x-1)}{x(x+2)}$$
Factorise everything first

$$= \frac{x+1}{x}$$
Cancel $(x+2)$ and $(x-1)$

Marks M1 $(x+1)(x-1)$ or $x(x+2)$ M1 Both factorised and a correct cancellation seen A1 $\frac{x+1}{x}$

10 Write $\frac{x+1}{2} - \frac{x-3}{5}$ as a single fraction in its simplest form.

Answer: $\frac{3x+11}{10}$

$$\frac{5(x+1)}{10} - \frac{2(x-3)}{10}$$
$$= \frac{5x+5-2x+6}{10}$$
$$= \frac{3x+11}{10}$$

Careful: $-2(x-3) = -2x+6$

Marks M1 Common denominator 10 M1 $5(x+1) - 2(x-3)$ A1 $\frac{3x+11}{10}$

Level 3 · Exam-style

17 marks

Hence, show that, dividing and solving equations, in real GCSE style.

11 Solve $\frac{x+2}{2} + \frac{x-1}{3} = 4$.

Answer: $x = 4$

$3(x+2) + 2(x-1) = 24$ *Multiply every term, including the 4, by 6*

$3x + 6 + 2x - 2 = 24$

$5x + 4 = 24$

$x = 4$

Marks M1 Multiplies every term by 6: $3(x+2) + 2(x-1) = 24$ M1 $5x + 4 = 24$ A1 $x = 4$

12 (a) Factorise $2x^2 - 5x - 3$.

Answer: $(2x+1)(x-3)$

$ac = -6; \quad (-6) \times 1 = -6,\ -6 + 1 = -5$

$2x^2 - 6x + x - 3 = 2x(x-3) + 1(x-3)$

$= (2x+1)(x-3)$

Marks M1 $(2x \pm 1)(x \pm 3)$ A1 $(2x+1)(x-3)$

(b) Hence simplify fully $\dfrac{2x^2-5x-3}{x^2-9}$.

Answer: $\dfrac{2x+1}{x+3}$

$\dfrac{(2x+1)(x-3)}{(x+3)(x-3)}$ — *Use part (a) for the top*

$= \dfrac{2x+1}{x+3}$ — *Cancel* $(x-3)$

Marks M1 $(x+3)(x-3)$ A1 $\dfrac{2x+1}{x+3}$ — ft their (a)

13 Solve $\dfrac{3}{x-1} - \dfrac{2}{x+1} = 1$.

Answer: $x = 3$ or $x = -2$

$3(x+1) - 2(x-1) = (x-1)(x+1)$ — *Multiply every term by* $(x-1)(x+1)$

$3x + 3 - 2x + 2 = x^2 - 1$

$x + 5 = x^2 - 1$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3$ or $x = -2$

Marks M1 Multiplies by $(x-1)(x+1)$: $3(x+1) - 2(x-1) = (x-1)(x+1)$ M1 $x + 5 = x^2 - 1$ M1 $x^2 - x - 6 = 0$ and $(x-3)(x+2) = 0$ A1 $x = 3$ and $x = -2$

14 Simplify fully $\dfrac{x^2-4}{3x} \div \dfrac{x+2}{6x^2}$.

Answer: $2x(x-2)$

$\dfrac{x^2-4}{3x} \times \dfrac{6x^2}{x+2}$ — *Dividing: flip the second fraction and multiply*

$= \dfrac{(x+2)(x-2)}{3x} \times \dfrac{6x^2}{x+2}$

$= (x-2) \times \dfrac{6x^2}{3x}$ — *Cancel* $(x+2)$

$= 2x(x-2)$ — $\dfrac{6x^2}{3x} = 2x$

Marks M1 $\dfrac{x^2-4}{3x} \times \dfrac{6x^2}{x+2}$ M1 $(x+2)(x-2)$ A1 $2x(x-2)$ — allow $2x^2 - 4x$

15 Show that $\frac{1}{x+1}+\frac{2}{x+3}$ can be written as $\frac{3x+5}{(x+1)(x+3)}$.

Answer: $\frac{(x+3)+2(x+1)}{(x+1)(x+3)}=\frac{3x+5}{(x+1)(x+3)}$

$$\frac{1(x+3)}{(x+1)(x+3)}+\frac{2(x+1)}{(x+1)(x+3)}$$
$$=\frac{x+3+2x+2}{(x+1)(x+3)}$$
$$=\frac{3x+5}{(x+1)(x+3)}$$

"Show that": every line must be written down

Marks M1 Common denominator $(x+1)(x+3)$ M1 $(x+3)+2(x+1)$ A1 Fully correct working to $\frac{3x+5}{(x+1)(x+3)}$

Level 4 · Challenge — 21 marks

Grade 8–9: combined operations, unknown coefficients and equations that lead to quadratics.

16 Simplify fully $\frac{x^2+2x-15}{2x^2-7x+3}\div\frac{x+5}{4x-2}$.

Answer: 2

$$\frac{x^2+2x-15}{2x^2-7x+3}\times\frac{4x-2}{x+5}$$
$$=\frac{(x+5)(x-3)}{(2x-1)(x-3)}\times\frac{2(2x-1)}{x+5}$$
$$=2$$

Dividing: flip and multiply

Factorise everything, including $4x-2$

$(x+5)$, $(x-3)$ *and* $(2x-1)$ *all cancel*

Marks M1 Flips the second fraction and multiplies M1 $(x+5)(x-3)$ and $(2x-1)(x-3)$ M1 $4x-2=2(2x-1)$ and correct cancelling A1 2

17 Solve $\frac{x}{x+2}+\frac{4}{x-1}=1$.

Answer: $x=-5$

$x(x-1)+4(x+2)=(x+2)(x-1)$ *Multiply every term by* $(x+2)(x-1)$

$x^2-x+4x+8=x^2+x-2$

$x^2+3x+8=x^2+x-2$

$2x=-10$ *The* x^2 *terms cancel, so this is linear*

$x=-5$ *Check:* $\frac{-5}{-3}+\frac{4}{-6}=\frac{5}{3}-\frac{2}{3}=1$

Marks M1 Multiplies by $(x+2)(x-1)$: $x(x-1)+4(x+2)=(x+2)(x-1)$ M1 $x^2+3x+8=x^2+x-2$ M1 $2x=-10$ A1 $x=-5$

18 Write $\frac{x+1}{x-2}-\frac{x-3}{x+2}$ as a single fraction in its simplest form.

Answer: $\frac{4(2x-1)}{(x-2)(x+2)}$

$\frac{(x+1)(x+2)-(x-3)(x-2)}{(x-2)(x+2)}$

$=\frac{x^2+3x+2-(x^2-5x+6)}{(x-2)(x+2)}$ *Keep the bracket: the minus applies to every term*

$=\frac{8x-4}{(x-2)(x+2)}$

$=\frac{4(2x-1)}{(x-2)(x+2)}$

Marks M1 Common denominator $(x-2)(x+2)$ M1 $(x+1)(x+2)-(x-3)(x-2)$ M1 $x^2+3x+2-(x^2-5x+6)$ A1 $\frac{8x-4}{(x-2)(x+2)}$ or $\frac{4(2x-1)}{x^2-4}$ oe

19 $\frac{x^2+ax-12}{x^2-9}$ simplifies to $\frac{x+4}{x+3}$. Find the value of a.

Answer: $a=1$

$x^2-9=(x+3)(x-3)$ *Factorise the bottom*

$\frac{x^2+ax-12}{(x+3)(x-3)}=\frac{x+4}{x+3}$ *So $(x-3)$ must have cancelled from the top*

$x^2+ax-12=(x+4)(x-3)=x^2+x-12$

$a=1$

Marks M1 $x^2-9=(x+3)(x-3)$ M1 The top must be $(x+4)(x-3)$ A1 $a=1$

20 (a) Show that the equation $\frac{5}{x-1}+\frac{2}{x+3}=1$ can be written as $x^2-5x-16=0$.

Answer: $5(x+3)+2(x-1)=(x-1)(x+3)$, so $7x+13=x^2+2x-3$, so $x^2-5x-16=0$.

$5(x+3)+2(x-1)=(x-1)(x+3)$ *Multiply every term by $(x-1)(x+3)$*

$7x+13=x^2+2x-3$

$0=x^2-5x-16$

Marks M1 Multiplies by $(x-1)(x+3)$: $5(x+3)+2(x-1)=(x-1)(x+3)$ M1 $7x+13=x^2+2x-3$ A1 Correct rearrangement to $x^2-5x-16=0$

(b) Hence solve $\frac{5}{x-1}+\frac{2}{x+3}=1$. Give your answers correct to 2 decimal places.

Answer: $x=7.22$ or $x=-2.22$

$x=\frac{5\pm\sqrt{(-5)^2-4\times1\times(-16)}}{2}=\frac{5\pm\sqrt{89}}{2}$ *It doesn't factorise, so use the formula*

$x=7.216\ldots$ or $x=-2.216\ldots$

$x=7.22$ or $x=-2.22$

Marks M1 Correct substitution: $\frac{5\pm\sqrt{25+64}}{2}$ A1 7.22 A1 -2.22